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Hamiltonian General Relativity in Finite Space and Cosmological Potential Perturbations

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Abstract

The Hamiltonian formulation of general relativity (GR) is considered in finite space-time and a specific reference frame given by the diffeo-invariant components of the Fock simplex in terms of the Dirac – ADM variables.

The evolution parameter and energy invariant with respect to the time-coordinate transformations are constructed by the separation of the cosmological scale factor $a(x^0)$ and its identification with the spatial averaging of the metric determinant, so that the dimension of the kinemetric group of diffeomorphisms coincides with the dimension of a set of variables whose velocities are removed by the Gauss-type constraints in accordance with the second Nöther theorem. This coincidence allows us to solve the energy constraint, fulfil Dirac's Hamiltonian reduction, and to describe the potential perturbations in terms of the Lichnerowicz scale-invariant variables distinguished by the absence of the time derivatives of the spatial metric determinant. It was shown that the Hamiltonian version of the cosmological perturbation theory acquires attributes of the theory of superfluid liquid, and it leads to a generalization of the Schwarzschild solution.

The astrophysical application of this approach to GR is considered under supposition that the Dirac – ADM Hamiltonian frame is identified with that of the Cosmic Microwave Background radiation distinguished by its dipole component in the frame of an Earth observer.

Contents

1. Introduction

In the year of celebration of the 90th anniversary of general relativity (GR) [1, 2] one can distinguish two treatments of general coordinate transformations i.) as generalization of the *frame transformations* and ii.) as *diffeomorphisms* of the GR action and a *geometric interval*. There is an essential difference between the frame group of the Lorentz – Poincaré-type [3] leading to a set of initial data and the *diffeomorphism group* of general coordinate transformations restricting these initial data by *constraints*. This difference was revealed by two Nöther theorems [4][1]. It became evident in the light of the formulation of GR in terms of the Fock simplex [7] defined as a diffeo-invariant Lorentz vector that helps us to separate diffeomorphisms from transformations of reference frames.

Just this separation of diffeomorphisms from the frame transformations is a starting position of this paper devoted to the consideration of GR in finite space-time in a specific reference frame given by the diffeo-invariant components of the Fock simplex in terms of the Dirac – ADM variables [8] widely used for the Hamiltonian formulation of GR. Fixing the "Hamiltonian frame" one can determine the group of diffeomorphisms of this frame.

Another starting position of the paper is the kinemetric group of diffeomorphisms of this "Hamiltonian frame" established in [9]. This kinemetric group contains global parametrizations of the coordinate time and local transformations of three spatial coordinates, and it requires to distinguish a set of variables with the same dimension, velocities (or momenta) of which are

[1] One of these theorems (the second) was formulated by Hilbert in his famous paper [2] (see also its revised version [5]). We should like to thank V.V. Nesterenko who draw our attention to this fact [6].

removed by the Gauss-type constraints from the phase space of diffeo-invariant physical variables in accordance with the second Nöther theorem.

A similar Hilbert-type [2, 5] *geometro*-dynamic formulation of special relativity (SR) [10, 11, 12] with reparameterizations of the coordinate evolution parameter shows that the energy constraint fixes a velocity of one of the dynamic variables that becomes a diffeo-invariant dynamic evolution parameter. In particular, in SR such a dynamic evolution parameter is well known, it is the fourth component of the Minkowskian space-time coordinate vector.

In order to realize a similar construction of the Dirac – ADM Hamiltonian GR [8], one should point out in GR a homogeneous variable that can be a diffeo-invariant evolution parameter in the field space of events [10] in accordance with the kinemetric diffeomorphism group of GR in the "Hamiltonian frame" [9]. The cosmological evolution is the irrefutable observational argument in favor of existence of such a homogeneous variable considered in GR as the cosmological scale factor.

The separation of the cosmological scale factor from metrics in GR is well-known as the cosmological perturbation theory proposed by Lifshits [13, 14] and applied as the basic tools for analysis of modern observational data in astrophysics and cosmology [15]. However, as it was shown in [16, 17], the Hamiltonian analysis of the standard Lifshits perturbation theory reveals that perturbation of the spatial metric determinant should be split from the cosmological scale factor by the projection operator onto nonhomogeneous class functions, otherwise the determinant perturbations contain one more variable equivalent to the scale factor that is the obstruction to the Hamiltonian approach.

Thus, in the Dirac approach to GR this homogeneous evolution parameter is not split; whereas the standard cosmological perturbation theory contains two such-type variables; so that in both the cases the dimension of the Hamiltonian constraints does not coincide with the dimension of the diffeomorphism group, which contradicts the second Nöther theorem.

In the present paper, in order to restore the number of variables of the initial GR, a homogeneous evolution parameter as a cosmological scale factor $a(x^0)$ is identified with the spatial averaging the metric determinant in the Dirac - ADM Hamiltonian reference frame with a finite space-time.

We show that this formulation of GR can solve the "energy-time" problem in both classical and quantum GR, and the latter has some attributes of the theory of superfluidity. In particular, there are spatial determinant free (i.e scale-invariant) variables introduced by Lichnerowicz [18, 19], in terms of which the classical GR contains only the Landau-type "friction-free" dynamics independent of the velocity-velocity interaction [20]. It leads to resolution of the energy constraint with respect to the cosmological scale momentum so that its positive and negative values become the generators of evolution of all scale-invariant field variables. The negative energy problem is solved by the primary and secondary quantization that leads to London's unique wave function [21] and Bogoliubov's squeezed condensate [22, 23], respectively. All these attributes are accompanied by a set of physical consequences that can be understood as quantum effects.

The diffeo-invariant Hamiltonian version of cosmological perturbation theory is applied in order to consider topical problems of modern cosmology by the low-energy decomposition of the reduced action in terms of the Lichnerowicz variables that identify conformal quantities with observables. It means that the Hubble law is explained by the evolution of masses, so that the Dark Energy chosen as the homogeneous free scalar field [24, 25, 26] is in agreement with primordial nucleosynthesis [27] and the last Supernova data [28, 29]. In this approach to GR and Standard Model (SM) all matter (including CMB radiation) appears as a final decay product of primordial vector W-, Z- bosons cosmologically created from the vacuum when their Compton length coincides with the universe horizon [30, 31]. The equations describing longitudinal vector

bosons in SM, in this case, become close to the equations of the Lifshits perturbation theory which are used in the classical inflationary model for description of the "power primordial spectrum" of the CMB radiation [32]. This means that the considered reference frame is identified with the CMB radiation one.

2. The Dirac – ADM Hamiltonian approach in terms of a simplex

2.1. GR in terms of Fock's simplex of reference

GR is given by two fundamental quantities: the *"dynamic"* action

$$S = \int d^4x\sqrt{-g}\left[-\frac{\varphi_0^2}{6}R(g) + \mathcal{L}_{(\mathrm{M})}\right] \equiv S_{\mathrm{GR}} + S_{\mathrm{M}}, \tag{1}$$

where $\varphi_0^2 = \frac{3}{8\pi}M^2_{\mathrm{Planck}} = \frac{3}{8\pi G}$, G is the Newton constant in the units $\hbar = c = 1$, $\mathcal{L}_{(\mathrm{M})}$ is the Lagrangian of the matter field, and a *"geometric interval"*

$$g_{\mu\nu}dx^\mu dx^\nu \equiv \omega_{(\alpha)}\omega_{(\alpha)} = \omega_{(0)}\omega_{(0)} - \omega_{(1)}\omega_{(1)} - \omega_{(2)}\omega_{(2)} - \omega_{(3)}\omega_{(3)}, \tag{2}$$

where $\omega_{(\alpha)}$ are linear differential forms as components of an orthogonal simplex of reference [7].

GR in terms of Fock's simplex contains two principles of relativity: the *"geometric"* — general coordinate transformations

$$x^\mu \to \tilde{x}^\mu = \tilde{x}^\mu(x^0, x^1, x^2, x^3), \qquad \omega_{(\alpha)}(x^\mu) \ \to\ \omega_{(\alpha)}(\tilde{x}^\mu) = \omega_{(\alpha)}(x^\mu) \tag{3}$$

and the set of transformations of a reference frame identified with the Lorentz transformations of an orthogonal simplex of reference

$$\omega_{(\alpha)} \ \to\ \overline{\omega}_{(\alpha)} = L_{(\alpha)(\beta)}\omega_{(\beta)}. \tag{4}$$

The invariance of the action with respect to frame transformations means that there are integrals of motion (the first Nöther theorem [4]); whereas the invariance of the action with respect to diffeomorphisms means that a part of degrees of freedom corresponds to pure gauge non-dynamical variables and diffeo-invariant potentials defined by constraints.

2.2. Diffeo-invariant variables and potentials

According to the second Nöther theorem [4], in any theory invariant with respect to diffeomorphisms there are constraints of velocities (momenta) that remove part of variables as pure gauge ones. These constraints are established in the specific reference frame distinguished by the Lorentz vector $l_\mu = (1, 0, 0, 0)$ [33].

In particular, in QED in the Minkowskian space-time given by equations $\partial_\mu F^{\mu\nu} = J^\nu$, the invariance of the theory with respect to diffeomorphisms well known as gauge transformations $A_\mu \to \widetilde{A}_\mu = A_\mu + \partial_\mu\lambda$, $\Psi \to \widetilde{\Psi} = e^{ie\lambda}\Psi$ allows us to remove the pure gauge longitudinal component, if $\lambda = \lambda^T$ is chosen so that it satisfies the equation $\triangle\lambda^T = \partial_k A_k$. After this transformation the zero component of the equations $\partial_\mu F^{\mu 0} = J^0$ known as the Gauss constraint takes the form $\triangle A_0^T = J_0^T$ (see [33]). Thus, all four components of the vector field $A_\mu = (A_0, A_k)$ can be split into the pure "gauge" longitudinal component $\partial_k A_k$, a diffeo-invariant potential $\triangle A_0^T = \triangle A_0 - \partial_k\partial_0 A_k$, and two diffeo-invariant transverse dynamic variables A_i^T $(\partial_k A_k^T \simeq 0)$. The potential equation is distinguished by the Laplacian $\triangle A_0^T$ without the time derivatives in contrast to the dynamic variable equation with d'Alambertian $\Box A_k^T$. Thus, the second Nöther theorem [4] means that diffeomorphisms lead to the first class constraints removing pure gauge velocities, whereas the corresponding pure gauge variables are removed by the second class constraints. There are similar problems in GR where a reference frame is given as a simplex.

2.3. The Dirac – ADM variables

The similar separation of diffeo-invariant dynamic metric components from potentials in GR is fulfilled in the specific reference frame in terms of the Dirac – ADM variables [8] defined as the Lichnerowicz transformation to the scale-invariant quantities $\omega^{(L)}_{(\mu)}$ [18, 19]

$$\omega_{(0)} = \psi^6 N_{\rm d} dx^0 \equiv \psi^2 \omega^{(L)}_{(0)}, \qquad \omega_{(b)} = \psi^2 \mathbf{e}_{(b)i}(dx^i + N^i dx^0) \equiv \psi^2 \omega^{(L)}_{(b)}; \tag{5}$$

here triads $\mathbf{e}_{(a)i}$ form the spatial metrics with $\det|\mathbf{e}| = 1$, $N_{\rm d}$ is the Dirac lapse function, N^i is shift vector and ψ is a determinant of the spatial metric.

The Hilbert action (1) in terms of the Dirac variables (5) takes the form

$$S_{\rm GR} = \int d^4x \left[\mathbf{K}[\varphi_0|e] - \mathbf{P}[\varphi_0|e] + \mathbf{S}[\varphi_0|e]\right], \tag{6}$$

where

$$\mathbf{K}[\varphi_0|e] \; = \; N_d \varphi_0^2 \left[-4{v_\psi}^2 + \frac{v^2_{(ab)}}{6}\right], \tag{7}$$

$$\mathbf{P}[\varphi_0|e] \; = \; \frac{N_d \varphi_0^2 \psi^7}{6}\left[{}^{(3)}R(\mathbf{e})\psi + 8\triangle\psi\right], \tag{8}$$

$$\mathbf{S}[\varphi_0|e] \; = \; 2\varphi_0^2 \left[\partial_0 v_\psi\right] - \partial_j \left[2\varphi^2 (N^j v_\psi) + \frac{\varphi_0^2}{3}\psi^2 \partial^j (\psi^6 N_d)\right], \tag{9}$$

are the kinetic, potential, and surface terms, respectively,

$$v_\psi \; = \; \frac{1}{N_d}\left[(\partial_0 - N^l \partial_l)\log\psi - \frac{1}{6}\partial_l N^l\right], \tag{10}$$

$$v_{(ab)} \; = \; \frac{1}{2}\left(\mathbf{e}_{(a)i} v^i_{(b)} + \mathbf{e}_{(b)i} v^i_{(a)}\right), \tag{11}$$

$$v_{(a)i} \; = \; \frac{1}{N_d}\left[(\partial_0 - N^l\partial_l)\mathbf{e}_{(a)i} + \frac{1}{3}\mathbf{e}_{(a)i}\partial_l N^l - \mathbf{e}_{(a)l}\partial_i N^l\right] \tag{12}$$

are velocities of the metric components, $\triangle\psi = \partial_i(\mathbf{e}^i_{(a)}\mathbf{e}^j_{(a)}\partial_j\psi)$ is the covariant Laplace operator, ${}^{(3)}R(\mathbf{e})$ is a three-dimensional curvature expressed in terms of triads $\mathbf{e}_{(a)i}$:

$${}^{(3)}R(\mathbf{e}) = -2\partial_i \left[\mathbf{e}^i_{(b)}\sigma_{(c)|(b)(c)}\right] - \sigma_{(c)|(b)(c)}\sigma_{(a)|(b)(a)} + \sigma_{(c)|(d)(f)}\sigma_{(f)|(d)(c)}. \tag{13}$$

Here

$$\sigma_{(a)|(b)(c)} = \mathbf{e}^j_{(c)}\nabla_i \mathbf{e}_{(a)k}\mathbf{e}^k_{(b)} = \frac{1}{2}\mathbf{e}_{(a)j}\left[\partial_{(b)}\mathbf{e}^j_{(c)} - \partial_{(c)}\mathbf{e}^j_{(b)}\right] \tag{14}$$

are the coefficients of the spin-connection (see [34] Eq. (98.9)), $\nabla_i \mathbf{e}_{(a)j} = \partial_i \mathbf{e}_{(a)j} - \Gamma^k_{ij}\mathbf{e}_{(a)k}$ are covariant derivatives, and $\Gamma^k_{ij} = \dfrac{1}{2}\mathbf{e}^k_{(b)}(\partial_i \mathbf{e}_{(b)j} + \partial_j \mathbf{e}_{(b)i})$.

The definition of canonical momenta

$$p_\psi \; = \; \frac{\partial \mathbf{K}[\varphi_0|e]}{\partial(\partial_0 \ln\widetilde{\psi})} = -8\varphi^2 v_\psi, \tag{15}$$

$$p^i_{(b)} \; = \; \frac{\partial \mathbf{K}[\varphi_0|e]}{\partial(\partial_0 \mathbf{e}_{(a)i})} = \mathbf{e}^i_{(a)}\frac{\varphi^2}{3}v_{(ab)} \tag{16}$$

allows us to represent the action (6) in the Hamiltonian form

$$S_{\rm GR} = \int dx^0 \int d^3x \left(\sum_F P_F \partial_0 F + \mathcal{C} - N_d T_0^0 \right), \tag{17}$$

where $P_F = (p_\psi, p^i_{(b)})$ are the set of momenta (15) – (16),

$$T_0^0 = \psi^7 \hat{\triangle}\psi + \sum_{I=0,8} \psi^I \tau_I, \tag{18}$$

is the sum of the Hamiltonian densities

$$\psi^7 \hat{\triangle}\psi \equiv \psi^7 \frac{4\varphi^2}{3} \partial_{(b)}\partial_{(b)}\psi, \tag{19}$$

$$\tau_{I=0} = \frac{6 p_{(ab)} p_{(ab)}}{\varphi^2} - \frac{16}{\varphi^2} p_\psi^2, \tag{20}$$

$$\tau_{I=8} = \frac{\varphi^2}{6} R^{(3)}(\mathbf{e}), \tag{21}$$

here $p_{(ab)} = \frac{1}{2}(\mathbf{e}^k_{(a)} p_{(b)k} + \mathbf{e}^k_{(b)} p_{(a)k})$, and

$$\mathcal{C} = N_{(b)} T^0_{(b)} + \lambda_0 p_\psi + \lambda_{(a)} \partial_k \mathbf{e}^k_{(a)} \tag{22}$$

is a set of the Lagrangian multipliers with Dirac constraints, including three first class constraints $T^0_{(a)} = 0$, where

$$T^0_{(a)} = T^0_i \mathbf{e}^i_{(a)} = -p_\psi \partial_{(a)}\psi + \frac{1}{6}\partial_{(a)}(p_\psi \psi) + 2 p_{(b)(c)} \sigma_{(b)|(a)(c)} - \partial_{(b)} p_{(b)(a)} \tag{23}$$

and fourth second class ones [8] $\partial_k \mathbf{e}^k_{(a)} = 0$, $p_\psi = 0$. The last constraint means the zero velocity of the spatial volume element

$$p_\psi = -8\varphi_0^2 v_\psi = 0 \to \partial_0(\psi^6) = \partial_l(\psi^6 N^l), \tag{24}$$

and it leads to the positive Hamiltonian density (20).

In this case, the equation of motion of the spatial determinant takes the potential form

$$7 N_d \psi^7 \hat{\triangle}\psi + \psi \hat{\triangle}[N_d \psi^7] + N_d \sum_{I=0,8} I \psi^I \tau_I = 0. \tag{25}$$

One can see that in a region of the space, where two dynamic variables are absent $\mathbf{e}_{(a)k} = \delta_{(a)k}$ (i.e. $\tau_{I=0,8} = 0$) there is the Schwarzschild-type solution of equations $\delta S/\delta N_d = -T^0_0 = 0$ and (25) that can be written in the form

$$\hat{\triangle}\psi = 0, \quad \hat{\triangle}[N_d\psi^7] = 0 \quad \to \quad \psi = 1 + \frac{r_g}{r}, \;\; [N_d \psi^7] = 1 - \frac{r_g}{r}, \;\; N^k = 0 \tag{26}$$

where r_g is the constant of the integration given by the boundary conditions.

One can see that the spatial coordinate diffeomorphisms $x_i \to \tilde{x}_i = \tilde{x}_i(x^0, x_1, x_2, x_3)$ can be used in order to fix three graviton momenta by the first class constraint (23) and remove the corresponding conjugate variables (i.e. three longitudinal gravitons) by the second class constraint $\partial_k \mathbf{e}^k_{(a)} = 0$ in complete correspondence with the application of the second Nöther theorem in QED [33] (see Section 2.2).

2.4. Problems of a diffeo-invariant evolution parameter

However, the time coordinate diffeomorphisms $x_0 \to \tilde{x}_0 = \tilde{x}_0(x^0, x_1, x_2, x_3)$ violate this QED/GR correspondence, because the time first class constraint $T_0^0 = 0$ fixes not velocity but variable ψ, and its velocity is removed by the second class constraint $p_\psi = 0$. Moreover, the Hamiltonian approach to GR is not invariant with respect to the time coordinate transformations. One can see that the solution (26) violates the symmetry of the interval (5) $\omega^{(L)}_{(0)} = \psi^4 N_d dx^0$ with respect to reparameterizations of the time-coordinate (3) $x^0 \ \to \ \widetilde{x}^0 = \widetilde{x}^0(x^0)$.

Zel'manov found [9] that a reference frame determined by forms (5) is invariant with respect to diffeomorphisms

$$x^0 \to \tilde{x}^0 = \tilde{x}^0(x^0); \qquad x_i \to \tilde{x}_i = \tilde{x}_i(x^0, x_1, x_2, x_3) \ , \tag{27}$$

$$\tilde{N}_d = N_d \frac{dx^0}{d\tilde{x}^0}; \qquad \tilde{N}^k = N^i \frac{\partial \tilde{x}^k}{\partial x_i}\frac{dx^0}{d\tilde{x}^0} - \frac{\partial \tilde{x}^k}{\partial x_i}\frac{\partial x^i}{\partial \tilde{x}^0} \ . \tag{28}$$

Only this group of transformations conserves a family of hypersurfaces $x^0 =$ const., and it is called the *"kinemetric"* subgroup of the group of general coordinate transformations. The *"kinemetric"* subgroup contains only homogeneous reparameterizations of the coordinate evolution parameter (x^0) and three local transformations of the spatial coordinates. This means that in finite space-time the diffeo-variant quantity (x^0) and the corresponding zero energy (18) are not observable.

We propose here to solve this problem as in [11, 12], where the frame (5) is redefined by pointing out diffeo-invariant homogeneous *"time-like variable"* in accordance with the dimension of the diffeomorphism group (27).

3. Diffeo-invariant formulation of GR

3.1. A homogeneous scale factor as evolution parameter

The Friedmann cosmology and the cosmological perturbation theory [13, 14] applied as the basic tools for analysis of modern observational data in astrophysics and cosmology [15] are the irrefutable arguments in favor of identification of such a diffeo-invariant homogeneous *"evolution parameter"* with the cosmological scale factor $a(x_0)$. This factor is introduced by the scale transformation of the metrics $g_{\mu\nu} = a^2(x^0)\widetilde{g}_{\mu\nu}$ and any field $F^{(n)}$ with the conformal weight (n): $F^{(n)} = a^n(x_0)\widetilde{F}^{(n)}$. In particular, the curvature

$$\sqrt{-g}R(g) = a^2\sqrt{-\widetilde{g}}R(\widetilde{g}) - 6a\partial_0\left[\partial_0 a\sqrt{-\widetilde{g}}\ \widetilde{g}^{00}\right] \tag{29}$$

can be expressed in terms of the new lapse function $\widetilde{N}_d$ and spatial determinant $\widetilde{\psi}$ in Eq. (5)

$$\widetilde{N}_d = [\sqrt{-\widetilde{g}}\ \widetilde{g}^{00}]^{-1} = a^2 N_d, \qquad \widetilde{\psi} = (\sqrt{a})^{-1}\psi. \tag{30}$$

In order to keep the number of variables, we identify $\log\sqrt{a}$ with the spatial volume "averaging" of $\log\psi$

$$\log\sqrt{a} = \langle\log\psi\rangle \equiv \frac{1}{V_0}\int d^3x \log\psi, \tag{31}$$

where $V_0 = \int d^3x < \infty$ is a finite volume. In this case, the new determinant variable $\widetilde{\psi}$ should be given in the orthogonal class of functions satisfying the identity

$$\int d^3x \log\widetilde{\psi} = \int d^3x\,[\log\psi - \langle\log\psi\rangle] \equiv 0. \tag{32}$$

One can call these functions the "deviations". The operation of "deviation" $\Pi_{\rm de}\cdot\mu=\overline{\mu}=\mu-\langle\mu\rangle$ is orthogonal to the operation of "averaging" $\Pi_{\rm av}\cdot\mu=\langle\mu\rangle$. The sum of these two operations is equal to unity $\Pi_{\rm de}\cdot\mu+\Pi_{\rm av}\cdot\mu=I\mu$, and square of these operations give them again: $\Pi_{\rm de}^2=\Pi_{\rm de}$ and $\Pi_{\rm av}^2=\Pi_{\rm av}$. One can see that the operations of "averaging" $\langle\mu\rangle$ and "deviation" $\overline{\mu}=\mu-\langle\mu\rangle$ have properties of projection operators.

Therefore, the variation of any functional of a "deviation" $\overline{\mu}=\mu-\langle\mu\rangle$ $S[\overline{\mu}]=S[\Pi_{\rm de}\cdot\mu]$ with respect to the "deviation"

$$\frac{\delta S[\Pi_{\rm de}\cdot\mu]}{\delta(\Pi_{\rm de}\cdot\mu)}=\Pi_{\rm de}\cdot\left[\frac{\delta S[\mu]}{\delta\mu}\right] \tag{33}$$

is a "deviation".

After the scale transformation (29), (30) action (1) takes the form

$$S[\varphi_0]=\widetilde{S}[\varphi]-\int dx^0(\partial_0\varphi)^2\int\frac{d^3x}{\widetilde{N}_d}; \tag{34}$$

here $\widetilde{S}[\varphi]$ is the action (1) in terms of metrics $\widetilde{g}$, where φ_0 is replaced by the running scale $\varphi(x^0)=\varphi_0 a(x^0)$ of all masses of the matter fields.

The energy constraint

$$\frac{\delta S[\varphi_0]}{\delta\widetilde{N}_d}=-T_0^0=\frac{(\partial_0\varphi)^2}{\widetilde{N}_d^2}-\widetilde{T}_0^0=0 \tag{35}$$

takes the algebraic form [11], where

$$\widetilde{T}_0^0\equiv-\frac{\delta\widetilde{S}[\varphi]}{\delta\widetilde{N}_d} \tag{36}$$

is the local energy density. This equation has the exact solution in both the local sector

$$N_{\rm inv}=\langle(\widetilde{N}_d)^{-1}\rangle\widetilde{N}_d=\frac{\left\langle\sqrt{\widetilde{T}_0^0}\right\rangle}{\sqrt{\widetilde{T}_0^0}}, \tag{37}$$

and the homogeneous one

$$\left[\frac{d\varphi}{d\zeta}\right]^2\equiv\varphi'^2=\rho_{(0)}\equiv\left\langle\sqrt{\widetilde{T}_0^0}\right\rangle^2, \tag{38}$$

where

$$\zeta(\varphi_0|\varphi)\equiv\int dx^0\langle(\widetilde{N}_d)^{-1}\rangle^{-1}=\pm\int_{\varphi}^{\varphi_0}\frac{d\widetilde{\varphi}}{\left\langle\sqrt{\widetilde{T}_0^0(\widetilde{\varphi})}\right\rangle} \tag{39}$$

is a diffeo-invariant time-coordinate and $N_{\rm inv}$ is the diffeo-invariant lapse function.

One can see that in the diffeo-invariant formulation of GR there is the almost complete QED/GR correspondence of the application of the second Nöther theorem, because the energy constraint (38) fixes the homogeneous velocity of the cosmic evolution. This fixation can be treated as the Hubble law in cosmology. The scale factor can be removed from the reduced phase space of diffeo-invariant variables, but not from measurable quantities, by the Hamiltonian reduction.

In Appendix A it was shown that the interactions with matter in terms of the scale-invariant Lichnerowicz variables $F^{(n)}_{(L)}=\psi^{-2n}F^{(n)}$, where n is the conformal weight, do not contain the time derivatives of the spatial determinant.

3.2. The diffeo-invariant Hamiltonian formulation

In order to calculate the canonical momenta, let us write the Hilbert action (1) in terms of the new Dirac variables (30)

$$S_{\rm GR}[g=a^2\widetilde{g}]=\int d^4x\,[\mathbf{K}[\varphi|e]-\mathbf{P}[\varphi|e]+\mathbf{S}[\varphi|e]]-\int dx^0\frac{(\partial_0\varphi)^2}{\widetilde{N}_d}=\int dx^0 L_{\rm GR}, \tag{40}$$

where

$$\mathbf{K}[\varphi|e] = \widetilde{N}_d\varphi^2\left[-4\overline{v}^2+\frac{v^2_{(ab)}}{6}\right], \tag{41}$$

$$\mathbf{P}[\varphi|e] = \frac{\widetilde{N}_d\varphi^2\widetilde{\psi}^7}{6}\left[{}^{(3)}R(\mathbf{e})\widetilde{\psi}+8\triangle\widetilde{\psi}\right], \tag{42}$$

$$\mathbf{S}[\varphi|e] = 2\varphi^2\,[\partial_0\overline{v}]-\partial_j\left[2\varphi^2(N^j\overline{v})+\frac{\varphi^2}{3}\widetilde{\psi}^2\partial^j(\widetilde{\psi}^6N_d)\right], \tag{43}$$

are the kinetic, potential, and "quasi-surface" terms, respectively,

$$\overline{v} = \frac{1}{\widetilde{N}_d}\left[(\partial_0-N^l\partial_l)\log\widetilde{\psi}-\frac{1}{6}\partial_lN^l\right], \tag{44}$$

$v_{(ab)}$ are velocities of the metric components given by Eqs. (11) and (12), $\triangle\psi=\partial_i(\mathbf{e}^i_{(a)}\mathbf{e}^j_{(a)}\partial_j\psi)$ is the covariant Laplace operator, ${}^{(3)}R(\mathbf{e})$ is a three-dimensional curvature expressed in terms of triads $\mathbf{e}_{(a)i}$ (13).

One can see that the Lagrangian in the action (40) includes three terms describing the spatial metric determinant

$$L_{\rm GR}=\int d^3x\mathcal{L}_{\rm GR}=-\int d^3x\overline{N_d}\left[4\varphi^2\;(\overline{v})^2+4\varphi\;\frac{\partial_0\varphi}{\widetilde{N}_d}\;\overline{v}+\left(\frac{\partial_0\varphi}{\widetilde{N}_d}\right)^2\right]+..., \tag{45}$$

where the first term arises from $\mathbf{K}$ (41), the second one (i.e. the velocity interaction) goes from the first term in $\mathbf{S}$ (43), and the third term goes from the scale factor term in Eq. (40).

Keeping the number of variables $\langle\log\widetilde{\psi}\rangle\equiv 0$ (32) and their velocities

$$\langle\overline{v}\rangle\equiv 0 \tag{46}$$

means that both $\log\widetilde{\psi}$ and $\overline{v}$ are given in the class of "deviation" functions distinguished by the projection operator $F=\overline{F}-\langle F\rangle$. In this class of functions the second term in Eq. (45) disappears. In this case momenta (54) and $\overline{p_\psi}$ become

$$P_\varphi\equiv\frac{\partial L_{\rm GR}}{\partial(\partial_0\varphi)}=-\int d^3x2\frac{\partial_0\varphi}{\widetilde{N}_d}=-2V_0\varphi' \tag{47}$$

and

$$\overline{p_\psi}\equiv\frac{\partial\mathcal{L}_{\rm GR}}{\partial(\partial_0\log\widetilde{\psi})}=-8\varphi^2\overline{v}=-\frac{8\varphi^2}{\widetilde{N}_d}\left[(\partial_0-N^l\partial_l)\log\widetilde{\psi}-\frac{1}{6}\partial_lN^l\right]. \tag{48}$$

All velocities are expressed in terms of canonical momenta, so that the Dirac Hamiltonian approach becomes consistent (see Appendix A).

In the diffeo-invariant version of GR in finite space-time one can express the action in Hamiltonian form in terms of momenta (47), (48) P_φ and $P_F = [\overline{p_\psi}, p^i_{(a)}, p_f]$

$$S[\varphi_0] = \int dx^0 \left[\int d^3x \left(\sum_F P_F \partial_0 F + C - \widetilde{N}_d \widetilde{T}^0_0\right) - P_\varphi \partial_0 \varphi + \frac{P^2_\varphi}{4\int d^3x (\widetilde{N}_d)^{-1}}\right], \qquad (49)$$

where $\mathcal{C} = N^i T^0_i + C_0 \overline{p_\psi} + C_{(a)} \partial_k \mathbf{e}^k_{(a)}$ is the sum of constraints with the Lagrangian multipliers $N^i, C_0,\ C_{(a)}$ and the energy–momentum tensor components T^0_i; these constraints include the transversality $\partial_i \mathbf{e}^i_{(a)} \simeq 0$ and the Dirac minimal surface [8]

$$\overline{p_\psi} \simeq 0 \qquad \Rightarrow \qquad \partial_j(\widetilde{\psi}^6 \mathcal{N}^j) = (\widetilde{\psi}^6)' \qquad (\mathcal{N}^j = N^j \langle N_d^{-1} \rangle). \qquad (50)$$

The first class constraints including three local ones (25) $T^i_0 = 0$ **and the Hamiltonian version of the homogeneous part of the energy constraint** (38)

$$P^2_\varphi = E^2_\varphi, \qquad (51)$$

where

$$E_\varphi = 2\int d^3x \sqrt{\widetilde{T}^0_0} = 2V_0 \left\langle \sqrt{\widetilde{T}^0_0} \right\rangle, \qquad (52)$$

fix three local longitudinal momenta and one homogeneous momentum P_φ, so that the dimension of the first class constraints coincides with the dimension of the kinemetric group of diffeomorphisms (27) of the Hamiltonian formulation of GR. The similar description of the first class constraint for the relativistic string is considered in Appendix B.

3.3. The Lifshits perturbation theory as an obstacle to Hamiltonian approach

Let us consider the definition of the deviation $\overline{v}$ in the class of function

$$\langle \overline{v} \rangle \neq 0, \qquad (53)$$

that does not keep the number of variable in GR. In this class of functions there is the coincidence of the homogeneous canonical momentum

$$P_\varphi \equiv \frac{\partial L_{\rm GR}}{\partial(\partial_0 \varphi)} = -\int d^3x \left[4\varphi\ \overline{v} + 2\frac{\partial_0 \varphi}{\widetilde{N}_d}\right] \qquad (54)$$

with the zero Fourier harmonics of $\overline{p_\psi} = \dfrac{\partial \mathcal{L}_{\rm GR}}{\partial(\partial_0 \log \widetilde{\psi})}$

$$\int d^3x \overline{p_\psi} \equiv \int d^3x \frac{\partial \mathcal{L}_{\rm GR}}{\partial(\partial_0 \log \widetilde{\psi})} = -2\varphi \int d^3x \left[4\varphi \overline{v} + 2\ \frac{\partial_0 \varphi}{\widetilde{N}_d}\ \right] \equiv 2\varphi P_\varphi. \qquad (55)$$

This means that the velocities could not be expressed in terms of canonical momenta and the system with the additional variable is not the Hamiltonian one. It is just the case of the standard cosmological perturbation theory [13, 14, 15] based on the interval

$$ds^2 = a^2(\eta)(1 + 2\Phi)d\eta^2 - a^2(\eta)(1 - 2\Psi)dx^i dx^j \qquad (56)$$

for which the Einstein equations

$$2R^\mu_\nu - \delta^\mu_\nu R = 4\pi G\ T^\mu_\nu \equiv t^\mu_\nu$$

take the form (see Eq.(4.15) in [15])

$$
\begin{aligned}
-3\mathcal{H}(\mathcal{H}\Phi+\Psi')+\triangle\Psi &= \delta t_{00} && (57)\\
3[(2\mathcal{H}'+\mathcal{H}^2)\Phi+\mathcal{H}\Phi'+\Psi''+2\mathcal{H}\Psi']+\triangle(\Phi-\Psi) &= \delta t_{ii}, && (58)
\end{aligned}
$$

where $\mathcal{H}=a'/a$, and $\delta t_{\mu\nu}$ are the perturbations. These equations follow from the variational principle, if the correspondent action of the type of (40) (considered in [15], see Eq. (10.7) p. 261) contains the velocity-velocity interaction. One can be convinced that in this case the canonical momenta $P_a \equiv \dfrac{\partial L_{GR}}{\partial(\partial_0 a)}$ and $P_\Psi \equiv \left\langle \dfrac{\partial \mathcal{L}_{GR}}{\partial(\partial_0\Psi)} \right\rangle$ coincide $2aP_a=P_\Psi$. The Hamiltonian approach is failure, as "velocities" $V_a=\partial_0 a$ and $V_\Psi=\langle\partial_0\Psi\rangle$ could not be expressed in terms of P_a, P_Ψ. The strong constraints $\langle\Psi\rangle\equiv 0$; $\langle\partial_0\Psi\rangle\equiv 0$ return us back to the Hamiltonian GR.

As we see below, the main differences of the "friction-free" version of GR from the Lifshits version [13, 15] are the potential perturbations of the scalar components $N_{\rm inv}, \widetilde{\psi}$ (given by Eqs. (70) – (78) in Section 4) instead of the kinetic ones and the nonzero shift vector $N^k \neq 0$ (determined by Eq. (62)). Recall that just the kinetic perturbations are responsible for the "primordial power spectrum" in the inflationary model [15]. The problem appears to describe CMBR by the potential perturbations.

3.4. The Hamiltonian reduction

One can find the evolution of all field variables $F(\varphi, x^i)$ with respect to φ by variation of the "reduced" action obtained as values of the Hamiltonian form of the initial action (49) onto the energy constraint (51)

$$
S[\varphi_0]_{P_\varphi=\pm E_\varphi} = \int\limits_{\varphi}^{\varphi_0} d\widetilde{\varphi} \left\{ \int d^3x \left[\sum_F P_F \partial_\varphi F + \bar{\mathcal{C}} \mp 2\sqrt{\widetilde{T}_0^0(\widetilde{\varphi})} \right] \right\}, \tag{59}
$$

where $\bar{\mathcal{C}}=\mathcal{C}/\partial_0\widetilde{\varphi}$ [11].

Here the reduced Hamiltonian function given by Eq. (52) can be treated as the "universe energy" by analogy with the "particle energy" in special relativity (SR). The reduced Hamiltonian $\sqrt{\widetilde{T}_0^0}$ is Hermitian, as the minimal surface constraint (50) removes a negative contribution of $\overline{p_\psi}$ from energy density. Thus, the diffeo-invariance gives us the solution of the problem of nonzero energy in GR by the Hamiltonian reduction like solution of a similar problem in SR.

One can see, that in the diffeo-invariant formulation of GR considered here the Hubble parameter becomes the generator of evolution with respect to the dynamic evolution parameter that abandons the reduced phase space but not the set of observables. The similar solution of the problem of the energy for the relativistic string is considered in Appendix B.

The main consequence of the separation of the cosmological scale factor is the globalization of the energy constraint (51). It fixes only the scale momentum $P_{\varphi_\pm}=\pm E_\varphi$, the values of which become the generator of evolution of all variables with respect to the evolution parameter φ [11] forward and backward, respectively. The negative energy problem can be solved by analogy with the modern quantum field theory as the primary quantization of the energy constraint $[P_\varphi^2-E_\varphi^2]\Psi_{\rm u}=0$ and the secondary quantization $\Psi_{\rm u}=(1/\sqrt{2E_\varphi})[A^+ + A^-]$ by the Bogoliubov transformation $A^+=\alpha B^+ + \beta^* B^-$, in order to diagonalize the equations of motion by the condensation of "universes" $<0|\frac{i}{2}[A^+A^+ - A^-A^-]|0>=R(\varphi)$ and describe cosmological creation of a "number" of universes $<0|A^+A^-|0>=N(\varphi)$ from the stable Bogoliubov vacuum $B^-|0>=0$ [23]. Vacuum postulate $B^-|0>=0$ leads to an arrow of the invariant time $\zeta\geq 0$ (39) and its

absolute point of reference $\zeta = 0$ at the moment of creation $\varphi = \varphi_I$; whereas the Planck value of the running mass scale $\varphi_0 = \varphi(\zeta = \zeta_0)$ belongs to the present day moment ζ_0.

The reduced action (59) shows us that the initial data at the beginning $\varphi = \varphi_I$ are independent of the present-day ones at $\varphi = \varphi_0$, therefore the proposal about an existence of the Planck epoch $\varphi = \varphi_0$ at the beginning [15] looks very doubtful.

4. The Diffeo-Invariant Scalar Potential Perturbations

4.1. The Hamiltonian perturbation theory

In diffeo-invariant formulation of GR in the specific reference frame the scalar potential perturbations can be defined as $N_{\rm int}^{-1} = 1+\overline{\nu}$ and $\widetilde{\psi} = e^{\overline{\mu}} = 1+\overline{\mu}+...$, where $\overline{\mu},\overline{\nu}$ are given in the class of functions distinguished by the projection operator $\overline{F} = F - \langle F\rangle$, so that $\langle\overline{F}\rangle \equiv 0$.

The explicit dependence of the metric simplex and the energy tensor $\widetilde{T}_0^0$ on $\widetilde{\psi}$ can be given in terms of the scale-invariant Lichnerowicz variables [18] introduced in Appendix A (A.4) and

$$\begin{aligned}\omega_{(0)}^{(L)} &= \widetilde{\psi}^4 N_{\rm int} d\zeta, \qquad \omega_{(b)}^{(L)} = \mathbf{e}_{(b)k}[dx^k + \mathcal{N}^k d\zeta], &(60)\\ \widetilde{T}_0^0 &= \widetilde{\psi}^7\hat{\triangle}\widetilde{\psi} + \sum_I \widetilde{\psi}^I a^{\frac{I}{2}-2}\tau_I, \qquad \tau_I \equiv \langle\tau_I\rangle + \overline{\tau_I}, &(61)\end{aligned}$$

where $\hat{\triangle}\widetilde{\psi} \equiv \dfrac{4\varphi^2}{3}\partial_{(b)}\partial_{(b)}\widetilde{\psi}$ is the Laplace operator and τ_I is partial energy density marked by the index I running a set of values $I = 0$ (stiff), 4 (radiation), 6 (mass), and 8 (curvature) in correspondence with a type of matter field contributions considered in Appendix A (A.22) – (A.25) (except of the Λ-term, $I = 12$). The negative contribution $-(16/\varphi^2)\overline{p_\psi}^2$ of the spatial determinant momentum in the energy density $\tau_{I=0}$ can be removed by the Dirac constraint [8] of the zero velocity of the spatial volume element (50)

$$\overline{p_\psi} = -8\varphi^2\frac{\partial_\zeta\widetilde{\psi}^6 - \partial_l[\widetilde{\psi}^6\mathcal{N}^l]}{\widetilde{\psi}^6 N_{\rm int}} = 0. \tag{62}$$

The diffeo-invariant part of the lapse function $N_{\rm int}$ is determined by the local part (37) of the energy constraint (35) that can be written as

$$\widetilde{T}_0^0 = N_{\rm int}^{-2}\rho_{(0)}, \qquad \to \qquad N_{\rm int}^{-1} = \sqrt{\widetilde{T}_0^0}\rho_{(0)}^{-1/2}, \tag{63}$$

where $\rho_{(0)} = \left\langle\sqrt{\widetilde{T}_0^0}\right\rangle^2$. In the class of functions $\overline{F} = F - \langle F\rangle$, the classical equation $\delta S/\delta\log\widetilde{\psi} = 0$ takes the form

$$\widetilde{N_d}\widetilde{\psi}\frac{\partial\widetilde{T}_0^0}{\partial\widetilde{\psi}} + \widetilde{\psi}\triangle\left[\frac{\partial\widetilde{T}_0^0}{\partial\triangle\widetilde{\psi}}\widetilde{N_d}\right] = 0.$$

Using the property of the deviation projection operator (33) $\delta S/\delta\overline{\mu} = \overline{D} = D - \langle D\rangle$, where $\overline{\mu} = \log\widetilde{\psi}$, we got the following equation

$$7N_{\rm inv}\widetilde{\psi}^7\hat{\triangle}\widetilde{\psi} + \widetilde{\psi}\hat{\triangle}[N_{\rm inv}\widetilde{\psi}^7] + N_{\rm inv}\sum_I I\widetilde{\psi}^I a^{\frac{I}{2}-2}\tau_I = \rho_{(1)}, \tag{64}$$

where $\rho_{(1)} = \left\langle 7N_{\rm inv}\widetilde{\psi}^7\hat{\triangle}\widetilde{\psi} + \widetilde{\psi}\hat{\triangle}[N_{\rm inv}\widetilde{\psi}^7] + \sum_I I\widetilde{\psi}^I a^{\frac{I}{2}-2}\tau_I\right\rangle$. Using (63) we can write for $\widetilde{\psi}$ a nonlinear equation

$$(\widetilde{T}_0^0)^{-1/2}\left[7\widetilde{\psi}^7\hat{\triangle}\widetilde{\psi} + \sum_I I\widetilde{\psi}^I a^{\frac{I}{2}-2}\tau_I\right] + \widetilde{\psi}\hat{\triangle}[(\widetilde{T}_0^0)^{-1/2}\widetilde{\psi}^7] = \rho_{(1)}\rho_{(0)}^{-1/2}. \tag{65}$$

In the infinite volume limit $\rho_{(n)} = 0,\ a = 1$ Eqs. (63) and (64) coincide with the equations of the diffeo-variant formulation of GR $T_0^0 = 0$ and (25) considered in Section 2.3.

For the small deviations $N_{\rm int}^{-1} = 1 + \overline{\nu}$ and $\widetilde{\psi} = e^{\overline{\mu}} = 1 + \overline{\mu} + ...$ the first orders of Eqs. (63) and (64) take the form

$$(-\hat{\triangle} - \rho_{(1)})\overline{\mu} \quad + \quad 2\rho_{(0)}\overline{\nu} \quad = \quad \overline{\tau}_{(0)}, \tag{66}$$

$$(14\hat{\triangle} + \rho_{(2)})\overline{\mu} \quad - \quad (\hat{\triangle} + \rho_{(1)})\overline{\nu} \quad = - \ \overline{\tau}_{(1)}, \tag{67}$$

where

$$\rho_{(n)} = \langle \tau_{(n)} \rangle \equiv \sum_I I^n a^{\frac{I}{2}-2} \langle \tau_I \rangle \tag{68}$$

$$\tau_{(n)} = \sum_I I^n a^{\frac{I}{2}-2} \tau_I. \tag{69}$$

The set of Eqs. (83) and (84) gives $\overline{\nu}$ and $\overline{\mu}$ in the form of a sum

$$\overline{\mu} = \frac{1}{14\beta} \int d^3y \left[D_{(+)}(x,y)\overline{T_{(+)}}(y) - D_{(-)}(x,y)\overline{T_{(-)}}(y) \right], \tag{70}$$

$$\overline{\nu} = \frac{1}{2\beta} \int d^3y \left[(1+\beta) D_{(+)}(x,y)\overline{T_{(+)}}(y) - (1-\beta) D_{(-)}(x,y)\overline{T_{(-)}}(y) \right], \tag{71}$$

where

$$\beta = \sqrt{1 + [\langle \tau_{(2)} \rangle - 14\langle \tau_{(1)} \rangle]/(98\langle \tau_{(0)} \rangle)}, \tag{72}$$

$$\overline{T_{(\pm)}} = (7\overline{\tau_{(0)}} - \overline{\tau_{(1)}}) \ \pm \ 7\beta\overline{\tau_{(0)}} \tag{73}$$

are the local currents, $D_{(\pm)}(x,y)$ are the Green functions satisfying the equations

$$[\pm \hat{m}^2_{(\pm)} - \hat{\triangle}] D_{(\pm)}(x,y) = \delta^3(x-y), \tag{74}$$

where $\hat{m}^2_{(\pm)} = 14(\beta \pm 1)\langle \tau_{(0)} \rangle \mp \langle \tau_{(1)} \rangle$.

The reduced Hamiltonian function (52) in terms of this solutions takes the form of the current-current interaction

$$E_\varphi = 2\int d^3x \sqrt{T_0^0} = 2V_0\sqrt{\langle \tau_{(0)} \rangle} + \tag{75}$$

$$+ \frac{1}{14\beta\sqrt{\langle \tau_{(0)} \rangle}} \int d^3x \int d^3y \left[\overline{T_{(+)}}(x) D_{(+)}(x,y) \overline{T_{(+)}}(y) + \overline{T_{(-)}}(x) D_{(-)}(x,y) \overline{T_{(-)}}(y) \right].$$

In the case of point mass distribution in a finite volume V_0 with the zero pressure and the density $\overline{\tau_{(1)}} = \dfrac{\overline{\tau_{(2)}}}{6} \equiv \sum\limits_J M_J \left[\delta^3(x-y_J) - \dfrac{1}{V_0} \right]$, solutions (70), (71) take a very important form

$$\overline{\mu}(x) = \sum_J \frac{r_{gJ}}{4r_J} \left[\gamma_1 e^{-m_{(+)}(z) r_J} + (1-\gamma_1)\cos m_{(-)}(z) r_J \right], \tag{76}$$

$$\overline{\nu}(x) = \sum_J \frac{2r_{gJ}}{r_J} \left[(1-\gamma_2) e^{-m_{(+)}(z) r_J} + \gamma_2 \cos m_{(-)}(z) r_J \right], \tag{77}$$

where

$$\gamma_1 = \frac{1+7\beta}{14\beta}, \quad \gamma_2 = \frac{(1-\beta)(7\beta-1)}{16\beta}, \quad r_{gJ} = \frac{3M_J}{4\pi\varphi^2}, \quad r_J = |x - y_J|, \quad m^2_{(\pm)} = \hat{m}^2_{(\pm)} \frac{3}{4\varphi^2}.$$

The minimal surface (50) $\partial_i[\overline{\psi}^6\mathcal{N}^i] - (\overline{\psi}^6)' = 0$ gives the shift of the coordinate origin in the process of evolution

$$\mathcal{N}^i = \left(\frac{x^i}{r}\right)\left(\frac{\partial_\zeta V}{\partial_r V}\right), \qquad V(\zeta, r) = \int\limits_0^r d\widetilde{r}\; \widetilde{r}^2\widetilde{\psi}^6(\zeta, \widetilde{r}). \tag{78}$$

In the infinite volume limit $\langle\tau_{(n)}\rangle = 0$ these solutions take the standard Newtonian form: $\overline{\mu} = D\cdot\tau_{(0)}$, $\overline{\nu} = D\cdot[14\tau_{(0)} - \tau_{(1)}]$, $\mathcal{N}^i = 0$ (where $\hat{\triangle}D(x) = -\delta^3(x)$).

4.2. Perturbation theory as generalization of Schwarzschild solution

One can see that another choice of variables for scalar potentials rearranges the perturbation series and leads to another result. In order to demonstrate this fact, let us choose the lapse function (37) as $N_{\rm inv}\widetilde{\psi}^7 = 1 - \overline{\nu_1}$ and keep $\widetilde{\psi} = 1 + \overline{\mu_1}$. In order to simplify equations of the scalar potentials $N_{\rm int}, \widetilde{\psi}$, one can introduce new table of symbols:

$$N_{\rm s} = \psi^7 N_{\rm inv}, \qquad T(\widetilde{\psi}) = \sum_I \widetilde{\psi}^{(I-7)} a^{\frac{I}{2}-2}\tau_I, \qquad \rho_{(0)} = \left\langle\sqrt{\widetilde{T}_0^0}\right\rangle^2 = \varphi'^2. \tag{79}$$

In terms of these symbols the action (49) can be presented as a generating functional of equations of the local scalar potentials $N_{\rm s}, \widetilde{\psi}$ and field variables F in terms of diffeo-invariant time ζ:

$$S[\varphi_0] = \int d\zeta \int d^3x \left[\sum_F P_F\partial_\zeta F + C_\zeta - N_{\rm s}\left(\hat{\triangle}\widetilde{\psi} + T(\widetilde{\psi})\right) - \frac{\widetilde{\psi}^7\rho_{(0)}}{N_{\rm s}}\right], \tag{80}$$

where $C_\zeta = C(dx^0/d\zeta)$.

The variations of this action with respect to $N_{\rm s}, \widetilde{\psi}$ lead to equations

$$\hat{\triangle}\widetilde{\psi} + T(\widetilde{\psi}) = \frac{\widetilde{\psi}^7\rho_{(0)}}{N_{\rm s}^2}, \tag{81}$$

$$\widetilde{\psi}\hat{\triangle}N_{\rm s} + N_{\rm s}\widetilde{\psi}\partial_{\widetilde{\psi}}T + 7\frac{\widetilde{\psi}^7\rho_{(0)}}{N_{\rm s}} = \rho_{(1)}, \tag{82}$$

respectively, we have used here the constraint (62) and the deviation projection operator (33) according to which $\rho_{(1)} = \langle\widetilde{\psi}\hat{\triangle}N_{\rm s} + N_{\rm s}\widetilde{\psi}\partial_{\widetilde{\psi}}T + 7\widetilde{\psi}^7\rho_{(0)}/N_{\rm s}\rangle$.

One can see that in the infinite volume limit $\rho_{(n)} = \langle\tau_I\rangle = 0$ Eqs. (81) and (82) reduce to the equations of the conventional GR with the Schwarzschild solutions $\overline{\psi} = 1 + \frac{r_g}{4r}$; $N_{\rm s} = 1 - \frac{r_g}{4r}$ in empty space, where Eqs. (81) and (82) become $\hat{\triangle}\overline{\psi} = 0$, $\hat{\triangle}N_{\rm s} = 0$.

For the small deviations $N_{\rm s} = 1 - \nu_1$ and $\widetilde{\psi} = 1 + \mu_1$ the first orders of Eqs. (81) and (82) take the form

$$[-\hat{\triangle} + 14\rho_{(0)} - \rho_{(1)}]\mu_1 + 2\rho_{(0)}\nu_1 = \overline{\tau}_{(0)} \tag{83}$$

$$[7\cdot 14\rho_{(0)} - 14\rho_{(1)} + \rho_{(2)}]\mu_1 + [-\hat{\triangle} + 14\rho_{(0)} - \rho_{(1)}]\nu_1 = 7\overline{\tau}_{(0)} - \overline{\tau}_{(1)}, \tag{84}$$

where

$$\rho_{(n)} = \langle\tau_{(n)}\rangle \equiv \sum_I I^n a^{\frac{I}{2}-2}\langle\tau_I\rangle. \tag{85}$$

This choice of variables determines $\overline{\mu_1}$ and $\overline{\nu_1}$ in the form of a sum

$$\widetilde{\psi} = 1 + \overline{\mu_1} \;=\; 1 + \frac{1}{2}\int d^3y \left[D_{(+)}(x,y)\overline{T}^{(\mu)}_{(+)}(y) + D_{(-)}(x,y)\overline{T}^{(\mu)}_{(-)}(y)\right], \tag{86}$$

$$N_{\rm inv}\widetilde{\psi}^7 = 1 - \overline{\nu_1} \;=\; 1 - \frac{1}{2}\int d^3y \left[D_{(+)}(x,y)\overline{T}^{(\nu)}_{(+)}(y) + D_{(-)}(x,y)\overline{T}^{(\nu)}_{(-)}(y)\right], \tag{87}$$

where β are given by Eqs. (72)

$$\overline{T}^{(\mu)}_{(\pm)} = \overline{\tau_{(0)}} \mp 7\beta[7\overline{\tau_{(0)}} - \overline{\tau_{(1)}}], \qquad \overline{T}^{(\nu)}_{(\pm)} = [7\overline{\tau_{(0)}} - \overline{\tau_{(1)}}] \pm (14\beta)^{-1}\overline{\tau_{(0)}} \tag{88}$$

are the local currents, $D_{(\pm)}(x,y)$ are the Green functions satisfying the equations (74) where $\hat{m}^2_{(\pm)} = 14(\beta \pm 1)\langle\tau_{(0)}\rangle \mp \langle\tau_{(1)}\rangle$. In the finite volume limit these solutions for $\widetilde{\psi}, N_{\rm inv}$ coincide with solutions (70) and (71), where $\overline{\nu_1} = \overline{\nu} - 7\overline{\mu}$ and $\overline{\mu_1} = \overline{\mu}$.

In the case of point mass distribution in a finite volume V_0 with the zero pressure and the density $\overline{\tau_{(0)}}(x) = \dfrac{\overline{\tau_{(1)}}(x)}{6} \equiv M\left[\delta^3(x-y) - \dfrac{1}{V_0}\right]$, solutions (86), (87) take a form

$$\widetilde{\psi} \;= 1 + \frac{r_g}{4r}\left[\gamma_1 e^{-m_{(+)}(z)r} + (1-\gamma_1)\cos m_{(-)}(z)r\right], \tag{89}$$

$$N_{\rm inv}\widetilde{\psi}^7 \;= 1 - \frac{r_g}{4r}\left[(1-\gamma_2) e^{-m_{(+)}(z)r} + \gamma_2\cos m_{(-)}(z)r\right], \tag{90}$$

where $\gamma_1 = \dfrac{1+7\beta}{2}$, $\gamma_2 = \dfrac{14\beta - 1}{28\beta}$, $r_g = \dfrac{3M}{4\pi\varphi^2}$, $r = |x-y|$. Both choices of variables (76), (77) and (89), (90) have spatial oscillations and the nonzero shift of the coordinate origin of the type of (78).

In the infinite volume limit $\langle\tau_{(n)}\rangle = 0$, $a = 1$ solutions (89) and (90) coincide with the isotropic version of the Schwarzschild solutions: $\widetilde{\psi} = 1 + \dfrac{r_g}{4r}$, $N_{\rm inv}\widetilde{\psi}^7 = 1 - \dfrac{r_g}{4r}$, $N^k = 0$. It is of interest to find an exact solution of Eq. (65) for different equations of state.

5. Cosmic Microwave Background Radiation

5.1. Status of SN data in terms of the scale-invariant variables

Einstein's correspondence principle [11] as the low-energy expansion of the *"reduced action"* (59) over the field density $T_{\rm s}$

$$2d\varphi\sqrt{T_0^0} = 2d\varphi\sqrt{\rho_0(\varphi) + T_{\rm s}} = d\varphi\left[2\sqrt{\rho_0(\varphi)} + T_{\rm s}/\sqrt{\rho_0(\varphi)}\right] + ...$$

gives the sum: $S^{(+)}|_{\rm constraint} = S^{(+)}_{\rm cosmic} + S^{(+)}_{\rm field} + \ldots$, where $S^{(+)}_{\rm cosmic}[\varphi_I|\varphi_0] = -2V_0\int\limits_{\varphi_I}^{\varphi_0} d\varphi\sqrt{\rho_0(\varphi)}$ is the reduced cosmological action (59), and

$$S^{(+)}_{\rm field} = \int\limits_{\eta_I}^{\eta_0} d\eta \int\limits_{V_0} d^3x\left[\sum_F P_F\partial_\eta F + \bar{\mathcal{C}} - T_{\rm s}\right] \tag{91}$$

is the standard field action in terms of the conformal time: $d\eta = d\varphi/\sqrt{\rho_0(\varphi)}$, in the conformal flat space–time with running masses $m(\eta) = a(\eta)m_0$.

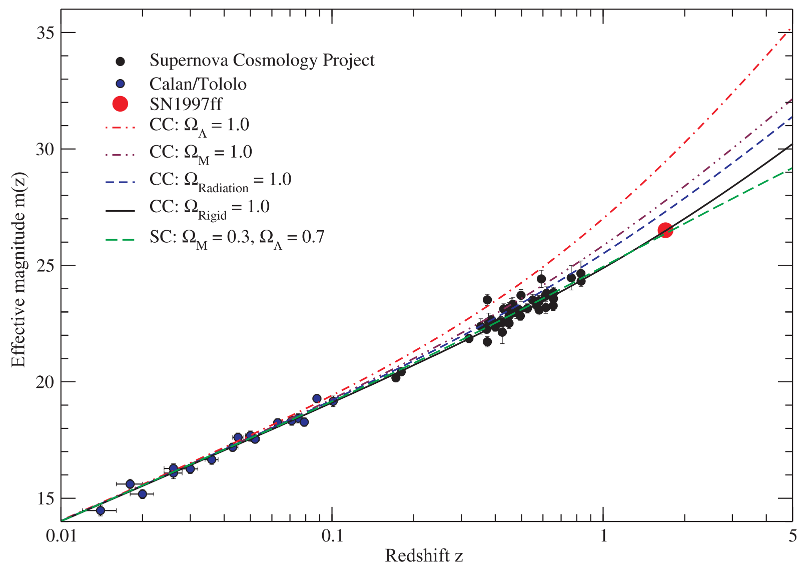

Figure 1: The Hubble diagram [25, 26] in cases of the *"scale-variant"* units of standard cosmology (SC) and the *"scale-invariant"* ones of conformal cosmology (CC). The points include 42 high-redshift Type Ia supernovae [28] and the reported farthest supernova SN1997ff [29]. The best fit to these data requires a cosmological constant $\Omega_\Lambda = 0.7$, $\Omega_{\rm CDM} = 0.3$ in the case of SC, whereas in CC these data are consistent with the dominance of the rigid (stiff) state.

This expansion shows us that the Hamiltonian approach in terms of the Lichnerowicz scale-invariant variables (60) and (A.4) identifies the "conformal quantities" with the observable ones including the conformal time $d\eta$, instead of $dt = a(\eta)d\eta$, the coordinate distance r, instead of Friedmann one $R = a(\eta)r$, and the conformal temperature $T_c = Ta(\eta)$, instead of the standard one T. Therefore, the scale-invariant variables distinguish the conformal cosmology (CC) [24, 35], instead of the standard cosmology (SC). In this case, the red shift of the spectral lines of atoms on cosmic objects

$$\frac{E_{\rm emission}}{E_0} = \frac{m_{\rm atom}(\eta_0 - r)}{m_{\rm atom}(\eta_0)} \equiv \frac{\varphi(\eta_0 - r)}{\varphi_0} = a(\eta_0 - r) = \frac{1}{1+z}$$

is explained by the running masses $m = a(\eta)m_0$ in action (91).

The conformal observable distance r loses the factor a, in comparison with the nonconformal one $R = ar$. Therefore, in the case of CC, the redshift – coordinate-distance relation $d\eta = d\varphi/\sqrt{\rho_0(\varphi)}$ corresponds to a different equation of state than in the case of SC [24]. The best fit to the data, including Type Ia supernovae [28, 29], requires a cosmological constant $\Omega_\Lambda = 0.7$, $\Omega_{\rm CDM} = 0.3$ in the case of the "scale-variant quantities" of standard cosmology. In the case of "conformal quantities" in CC, the Supernova data [28, 29] are consistent with the dominance of the stiff (rigid) state, $\Omega_{\rm Rigid} \simeq 0.85 \pm 0.15$, $\Omega_{\rm Matter} = 0.15 \pm 0.15$ [24, 25, 26]. If $\Omega_{\rm Rigid} = 1$, we have the square root dependence of the scale factor on conformal time $a(\eta) = \sqrt{1 + 2H_0(\eta - \eta_0)}$. Just this time dependence of the scale factor on the measurable time (here – conformal one) is used for description of the primordial nucleosynthesis [26, 27].

This stiff state is formed by a free scalar field when $E_\varphi = 2V_0\sqrt{\rho_0} = Q/\varphi$. In this case there is an exact solution of Bogoliubov's equations of the number of universes created from a vacuum with the initial data $\varphi(\eta = 0) = \varphi_I, H(\eta = 0) = H_I$ [23].

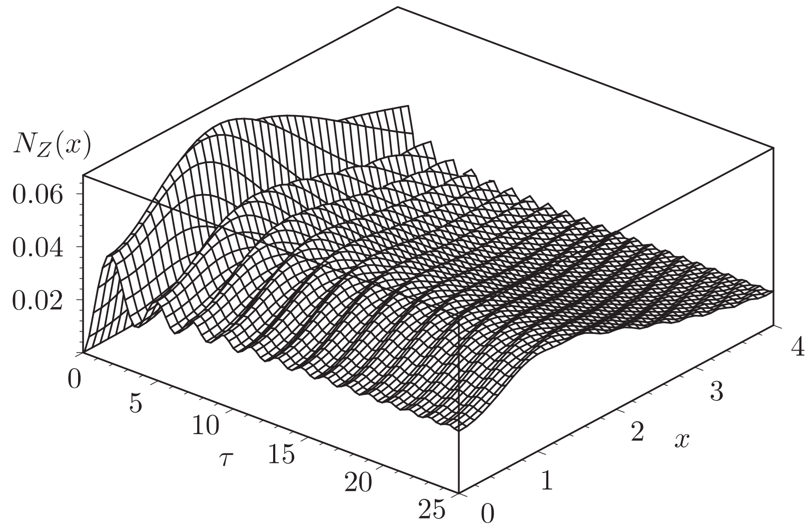

Figure 2: Longitudinal ($N_Z(x)$) components of the boson distribution versus the dimensionless time $\tau = 2\eta H_I$ and the dimensionless momentum $x = q/M_I$ at the initial data $M_I = H_I$ [30, 31].

5.2. Cosmological creation of matter

These initial data φ_I and H_I are determined by the parameters of matter cosmologically created from the Bogoliubov vacuum at the beginning of a universe $\eta \simeq 0$.

The Standard Model (SM) density $T_{\rm s}$ in action (91) shows us that W-, Z- vector bosons have maximal probability of this cosmological creation due to their mass singularity [30]. One can introduce the notion of a particle in a universe if the Compton length of a particle defined by its inverse mass $M_{\rm I}^{-1} = (a_{\rm I} M_{\rm W})^{-1}$ is less than the universe horizon defined by the inverse Hubble parameter $H_{\rm I}^{-1} = a_{\rm I}^2 (H_0)^{-1}$ in the stiff state. Equating these quantities $M_{\rm I} = H_{\rm I}$ one can estimate the initial data of the scale factor $a_{\rm I}^2 = (H_0/M_{\rm W})^{2/3} = 10^{-29}$ and the primordial Hubble parameter $H_{\rm I} = 10^{29} H_0 \sim 1\ {\rm mm}^{-1} \sim 3K$. Just at this moment there is an effect of intensive cosmological creation of the vector bosons described in [30, 31] (see Fig. 2); in particular, the distribution functions of the longitudinal vector bosons demonstrate us a large contribution of relativistic momenta. Their conformal (i.e. observable) temperature T_c (appearing as a consequence of collision and scattering of these bosons) can be estimated from the equation in the kinetic theory for the time of establishment of this temperature $\eta^{-1}_{relaxation} \sim n(T_c) \times \sigma \sim H$, where $n(T_c) \sim T_c^3$ and $\sigma \sim 1/M^2$ is the cross-section. This kinetic equation and values of the initial data $M_{\rm I} = H_{\rm I}$ give the temperature of relativistic bosons

$$T_c \sim (M_{\rm I}^2 H_{\rm I})^{1/3} = (M_0^2 H_0)^{1/3} \sim 3K \qquad (92)$$

as a conserved number of cosmic evolution compatible with the Supernova data [24, 28, 29]. We can see that this value is surprisingly close to the observed temperature of the CMB radiation $T_c = T_{\rm CMB} = 2.73$ K.

The primordial mesons before their decays polarize the Dirac fermion vacuum (as the origin of axial anomaly [36, 37, 38, 39]) and give the baryon asymmetry frozen by the CP – violation. The value of the baryon–antibaryon asymmetry of the universe following from this axial anomaly was estimated in [30] in terms of the coupling constant of the superweak-interaction

$$n_b/n_\gamma \sim X_{CP} = 10^{-9}. \qquad (93)$$

The boson life-times $\tau_W = 2H_I\eta_W \simeq (2/\alpha_W)^{2/3} \simeq 16,\ \tau_Z \sim 2^{2/3}\tau_W \sim 25$ determine the present-

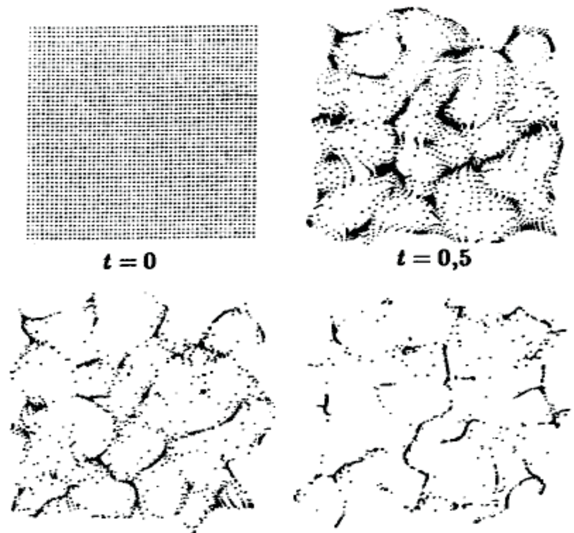

Figure 3: The diffusion of a system of particles moving in the space $ds^2 = d\eta^2 - (dx^i + N^i d\eta)^2$ with periodic shift vector N^i and zero momenta could be understood from analysis of O.D.E. $dx^i/d\eta = N^i$ considered in the two-dimensional case in [40], if we substitute $t = m_{(-)}\eta$ and $N^i \sim \frac{x^i}{r}\sin m_{(-)}r$ in the equations, where $m_{(-)}$ is defined by Eq. (77).

day visible baryon density

$$\Omega_b \sim \alpha_W = \alpha_{QED}/\sin^2\theta_W \sim 0.03. \tag{94}$$

All these results (92) – (94) testify to that all visible matter can be a product of decays of primordial bosons, and the observational data on CMBR can reflect parameters of the primordial bosons, but not the matter at the time of recombination. In particular, the length of the semi-circle on the surface of the last emission of photons at the life-time of W-bosons in terms of the length of an emitter (i.e. $M_W^{-1}(\eta_L) = (\alpha_W/2)^{1/3}(T_c)^{-1}$) is $\pi \cdot 2/\alpha_W$. It is close to $l_{min} \sim 210$ of CMBR, whereas $(\triangle T/T)$ is proportional to the inverse number of emitters $(\alpha_W)^3 \sim 10^{-5}$.

The temperature history of the expanding universe copied in the "conformal quantities" looks like the history of evolution of masses of elementary particles in the cold universe with the constant conformal temperature $T_c = a(\eta)T = 2.73$ K of the cosmic microwave background.

5.3. Large-scale structure of the matter distribution

In the contrast to standard cosmological perturbation theory [13, 14, 15] the diffeo-invariant version of the perturbation theory do not contain time derivatives that are responsible for the CMB "primordial power spectrum" in the inflationary model [15]. However, the diffeo-invariant version of the Dirac Hamiltonian approach to GR gives us another possibility to explain the CMBR "spectrum" and other topical problems of cosmology by cosmological creation of the vector bosons considered above. The equations describing the longitudinal vector bosons in SM, in this case, are close to the equations that follow from the Lifshits perturbation theory and are used, in the inflationary model, for description of the "power primordial spectrum" of the CMB radiation.

The next differences are a nonzero shift vector and spatial oscillations of the scalar potentials determined by $\hat{m}^2_{(-)}$ (see Fig. 3). In the scale-invariant version of cosmology [24], the SN data dominance of stiff state $\Omega_{\rm Stiff} \sim 1$ determines the parameter of spatial oscillations $\hat{m}^2_{(-)} = \dfrac{6}{7}H_0^2[\Omega_{\rm R}(z+1)^2 + \dfrac{9}{2}\Omega_{\rm Mass}(z+1)]$. The redshifts in the recombination epoch $z_r \sim 1100$ and the clustering parameter [40] $r_{\rm clustering} = \dfrac{\pi}{\hat{m}_{(-)}} \sim \dfrac{\pi}{H_0\Omega_R^{1/2}(1+z_r)} \sim 130\,{\rm Mpc}$ recently discovered in the researches of a large scale periodicity in redshift distribution [41, 42] lead to a reasonable value of the radiation-type density $10^{-4} < \Omega_R \sim 3\cdot 10^{-3} < 5\cdot 10^{-2}$ at the time of this epoch.

6. Conclusions

We supposed that the Universe was created in a specific reference frame, where the Hamiltonian approach to GR is constructed in the finite space-time with the diffeomorphisms keeping the frame of reference. This frame is remembered by the products of decay of the primordial massive vector bosons created from the Bogoliubov stable vacuum.

The physical content of the Universe is described by both the relativistic invariants (of the type of amplitudes of scattering) and relativistic covariant quantities like diffeo-invariant time, finite volume, temperature, density, etc. Therefore, in contrast to the S-matrix approach that depends on only relativistic invariants [43], for the complete description of the Universe we need a set of diffeo-invariant physical quantities changing under the Lorentz relativistic transformations of the type of the dipole component of the CMBR appearing in the frame of an Earth observer. Therefore, the quantum creation of the Universe in the finite space-time requires the separation of the transformations of frames from the diffeomorphisms in the context of the David Hilbert formulation of GR [2].

Moreover, just this separation (including the choice of an evolution parameter in GR as a cosmological scale factor) simplifies the Hamiltonian equations and leads to exact resolution of the energy constraint with respect to the canonical momentum of the scale factor. The positive and negative values of this momentum in the "field space of events" play the role of the generators of evolution forward and backward, respectively. These values of the momentum onto equations of motion can be called the "frame energies".

The solution of the problem of the "negative frame energy" by the primary quantization and the secondary one (on the analogy of the pathway passed by QFT in the 20th century) reveals in GR all attributes of the theory of superfluid quantum liquid: Landau-type absence of "friction", London-type WDW wave function, and Bogoliubov-type condensate of quantum universes.

The postulate of the quantum Bogoliubov vacuum as the state with the minimal "energy" leads to the absolute beginning of geometric time. The fundamental principle of the Hermitian Hamiltonian of the evolution in the field space of events keeps only the potential perturbations of the scalar metric components in contrast to the standard cosmological perturbation theory [13] keeping only the kinetic perturbations and the "friction" term in the action that is responsible for the "primordial power spectrum" in the inflationary model [15]. However, the Quantum Gravity considered as the theory of superfluidity gives us a possibility to explain this "spectrum" and other topical problems of cosmology by the cosmological creation of the primordial W-, Z- bosons from vacuum, when their Compton length coincides with the universe horizon.

The Einstein correspondence principle identifies the conformal quantities with the "measurable" ones, and the uncertainty principle establishes the point of the beginning of the cosmological creation of the primordial W-, Z- bosons from vacuum due to their mass singularity at the moment $a_I^2 \simeq 10^{-29}, H_I^{-1} \simeq 1$ mm. The equations describing the longitudinal vector bosons in SM,

in this case, are close to the equations of the inflationary model used for description of the "power primordial spectrum" of the CMB radiation. We listed the set of theoretical and observational arguments in favor of that the CMB radiation can be a final product of primordial vector W-, Z- bosons cosmologically created from the vacuum.

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Appendix A: The energy density in the massive electrodynamics

As the model of the matter let us consider massive electrodynamics in GR

$$S = \int d^4x\sqrt{-g}\left[-\frac{\varphi_0^2}{6}R(g) + \mathcal{L}_{\rm m}\right], \tag{A.1}$$

where $\mathcal{L}_{\rm m}$ is the Lagrangian of the massive vector and spinor fields

$$\mathcal{L}_{\rm m} = -\frac{1}{4}F_{\mu\nu}F_{\alpha\beta}g^{\mu\alpha}g^{\nu\beta} - M_0^2 A_\mu A_\nu g^{\mu\nu} - \widetilde{\psi} i\gamma^\sigma(D_\sigma - ieA_\sigma)\Psi - m_0\widetilde{\psi}\hat{\Psi} \tag{A.2}$$

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the stress tensor,

$$D_\delta = \partial_\delta - i\frac{1}{2}[\gamma_{(\alpha)}\gamma_{(\beta)}]\sigma_{\delta(\alpha)(\beta)}, \tag{A.3}$$

is the Fock covariant derivative [7], $\gamma_{(\beta)} = \gamma^\mu e_{(\beta)\mu}$ are the Dirac γ-matrices, summed with tetrads $e_{(\beta)\nu}$, and $\sigma_{\sigma(\alpha)(\beta)} = e^\nu_{(\beta)}(\nabla_\mu e_{(\alpha)\nu})$ are coefficients of spin-connection [7, 34].

The Lagrangian of the massive fields (A.2) can be rewritten in terms of the Lichnerowicz variables

$$A^L{}_\mu = A_\mu, \qquad \Psi^L = a^{3/2}\psi^3\Psi, \tag{A.4}$$

that lead to fields with masses depending on the scale factor $a\psi^2$

$$m_{(\rm L)} = m_0 a\psi^2 = m\psi^2, \qquad M_{(\rm L)} = M_0 a\psi^2 = M\psi^2. \tag{A.5}$$

These fields are in the space defined by the component of the frame

$$\omega^{(\rm L)}_{(0)} = \widetilde{\psi}^4\ \widetilde{N}_d dx^0, \tag{A.6}$$

$$\omega^{(\rm L)}_{(a)} = \mathbf{e}_{(a)i}(dx^i + N^i dx^0). \tag{A.7}$$

with the unit metric determinant $|\mathbf{e}| = 1$.

As the result, the Lagrangian of the matter fields (A.2) takes the form

$$\begin{aligned}\sqrt{-g}\mathcal{L}_{\rm m}(A,\widetilde{\psi},\Psi) &= \frac{1}{i}\widetilde{\psi}^L\gamma^0\left(\partial_0 - N^k\partial_k + \frac{1}{2}\partial_l N^l - ieA_0\right)\Psi^L - \widetilde{N}_d\mathcal{H}_\Psi + \\ &+\widetilde{N}_d\left[-J_{5(c)}v_{[ab]}\varepsilon_{(c)(a)(b)} + \frac{\widetilde{\psi}^4}{2}\left(v_{i(\rm A)}v^i_{(\rm A)} - \frac{1}{2}F_{ij}F^{ij}\right) - \widetilde{\psi}^8 M^2 A^2_{(b)} - \frac{\pi_0^2}{M^2}\right] - \\ &+\widetilde{N}_d\ \widetilde{\psi}^6 m\widetilde{\psi}^L\Psi^L - \pi_0[N^i A_i - A_0]\ ,\end{aligned} \tag{A.8}$$

where the Legendre transformation $A_0^2/(2\widetilde{N}_d) = \pi_0 A_0 - \widetilde{N}_d \pi_0^2/2$ with the subsiduary field π_0 is used for linearizing the massive term;

$$\mathcal{H}_\Psi = -\widetilde{\psi}^4[i\widetilde{\psi}^L \gamma_{(b)} D_{(b)} \Psi^L + J_5^0 \sigma - \partial_k J^k] \tag{A.9}$$

is the Hamiltonian density of the fermions,

$$\begin{aligned}
v_{[ab]} &= \frac{1}{2}\left(\mathbf{e}_{(a)i} v^i_{(b)} - \mathbf{e}_{(b)i} v^i_{(a)}\right), && \text{(A.10)}\\
D_{(b)}\Psi^L &= [\partial_{(b)} - \frac{1}{2}\partial_k \mathbf{e}^k_{(b)} - ieA_{(b)}]\Psi^L, && \text{(A.11)}\\
v_{i(\mathrm{A})} &= \frac{1}{\psi^4 \widetilde{N}_d}[\partial_0 A_i - \partial_i A_0 + F_{ij}N^j] && \text{(A.12)}
\end{aligned}$$

are the field velocities, and

$$J_{5(c)} = \frac{i}{2}(\widetilde{\psi}^L \gamma_5 \gamma_{(c)} \Psi^L)\ , \qquad J_5^0 = \frac{i}{2}(\bar{\Psi}^L \gamma_5 \gamma^0 \Psi^L), \qquad J_k = \frac{i}{2}\bar{\Psi}^L \gamma_k \Psi^L\ ; \tag{A.13}$$

are the currents, $\sigma = \sigma_{(a)(b)|(c)}\varepsilon_{(a)(b)(c)}$, where $\varepsilon_{(a)(b)(c)}$ denotes the Levi-Civita tensor.

The canonical conjugated momenta take the form

$$\begin{aligned}
P_\varphi &= -2V_0 \frac{\partial_0 \varphi}{N_0} && = -2V_0 \frac{d\varphi}{d\zeta} \equiv -2V_0 \varphi' && \text{(A.14)}\\
\overline{p_\psi} &= \frac{\partial \mathbf{K}[\varphi|e]}{\partial(\partial_0 \ln \widetilde{\psi})} && = -8\varphi^2 \overline{v}, && \text{(A.15)}\\
p^i_{(b)} &= \frac{\partial[\mathbf{K}[\varphi|e] + \sqrt{-g}\mathcal{L}_\mathrm{m}]}{\partial(\partial_0 \mathbf{e}_{(a)i})} && = \mathbf{e}^i_{(a)}\left[\frac{\varphi^2}{3} v_{(ab)} - J_{5(c)}\varepsilon_{(c)(a)(b)}\right], && \text{(A.16)}\\
P^i_{(\mathrm{A})} &= \frac{\partial[\sqrt{-g}\mathcal{L}_\mathrm{m}]}{\partial(\partial_0 A_i)} && = \widetilde{\psi}^4 v^i_{(\mathrm{A})}, && \text{(A.17)}\\
P_{(\Psi)} &= \frac{\partial[\sqrt{-g}\mathcal{L}_\mathrm{m}]}{\partial(\partial_0 \Psi^L)} && = \frac{1}{i}\widetilde{\psi}^L \gamma^0. && \text{(A.18)}
\end{aligned}$$

Then, the action (A.1) one can be represented in the Hamiltonian form

$$S = \int dx^0 \left[-P_\varphi \partial_0 \varphi + N_0 \frac{P_\varphi^2}{4V_0} + \int d^3x \left(\sum_F P_F \partial_0 F + \mathcal{C} - \widetilde{N}_d T^0_{0t}\right)\right], \tag{A.19}$$

where P_F is a set of the field momenta (A.15) – (A.18),

$$T^0_{0t} = \widetilde{\psi}^7 \hat{\triangle} \widetilde{\psi} + \sum_{I=0,4,6,8} \widetilde{\psi}^I \tau_I, \tag{A.20}$$

is the sum of the Hamiltonian densities including the gravity density

$$\begin{aligned}
\widetilde{\psi}^7 \hat{\triangle} \widetilde{\psi} &\equiv \widetilde{\psi}^7 \frac{4\varphi^2}{3} \partial_{(b)} \partial_{(b)} \widetilde{\psi}, && \text{(A.21)}\\
\tau_{I=0} &= \frac{6\widetilde{p}_{(ab)}\widetilde{p}_{(ab)}}{\varphi^2} - \frac{16}{\varphi^2}\overline{p_\psi}^2 + \frac{\pi_0^2}{2a^2M^2}, && \text{(A.22)}\\
\tau_{I=4} &= \frac{P_{i(\mathrm{A})}P^i_{(\mathrm{A})} + F_{ij}F^{ij}}{2} - [i\widetilde{\psi}^L \gamma_{(b)} D_{(b)} \Psi^L + J_5^0 \sigma - \partial_k J^k], && \text{(A.23)}\\
\tau_{I=6} &= m\widetilde{\psi}^L \Psi^L, && \text{(A.24)}\\
\tau_{I=8} &= \frac{\varphi^2}{6} R^{(3)}(\mathbf{e}) + \frac{M^2 A^2_{(b)}}{2}, && \text{(A.25)}
\end{aligned}$$

here $\widetilde{p}_{(ab)} = \frac{1}{2}(\mathbf{e}^i_{(a)}\widetilde{p}_{(b)i} + \mathbf{e}^i_{(b)}\widetilde{p}_{(a)i})$, $\widetilde{p}_{(b)i} = p_{(b)i} + \mathbf{e}^i_{(a)}\varepsilon_{(c)(a)(b)}J_{(c)}$,

$$\mathcal{C} = A_0[\partial_i P^i_{(A)} + eJ_0 - \pi_0] + N_{(b)}T^0_{(b)t} + \lambda_0\overline{p_\psi} + \lambda_{(a)}\partial_k\mathbf{e}^k_{(a)} \tag{A.26}$$

denotes the sum of the constraints, where $J_0 = \widetilde{\psi}^L\gamma_0\Psi^L$ is the zero component of the current; $A_0, N_d, N^i, \lambda_0, \lambda_{(a)}$ are the Lagrange multipliers including the Dirac condition of the minimal 3-dimensional hyper-surface [8]

$$p_{\widetilde{\psi}} = \overline{v} = 0 \to (\partial_0 - N^l\partial_l)\log\widetilde{\psi} = \frac{1}{6}\partial_l N^l, \tag{A.27}$$

that gives a positive value of the Hamiltonian density (A.22), and

$$\begin{aligned} T^0_{(a)t} = & - \overline{p_\psi}\partial_{(a)}\widetilde{\psi} + \frac{1}{6}\partial_{(a)}(\overline{p_\psi}\widetilde{\psi}) + 2p_{(b)(c)}\gamma_{(b)|(a)(c)} - \partial_{(b)}p_{(b)(a)} + \\ & - \frac{1}{i}\bar{\Psi}^I\gamma^0\partial_{(a)}\Psi^I - \frac{1}{2i}\partial_{(a)}\left(\bar{\Psi}^I\gamma^0\Psi^I\right) - P^i_{(\mathrm{A})}F_{ik}\mathbf{e}^k_{(a)} - \pi_0 A_{(a)}. \end{aligned} \tag{A.28}$$

are the components of the total energy-momentum tensor $T^0_{(a)} = T^0_i\mathbf{e}^i_{(a)}$.

Appendix B: Diffeo-Invariant Content of a Relativistic String

To illustrate the invariant Hamiltonian reduction by putting strong constraints like (32) let us consider the action for a relativistic string [44] in the form which has been done by L. Brink, P. Di Vecchia, P. Howe [45]

$$S = -\frac{\gamma}{2}\int\int d^2u\sqrt{-g}g^{\alpha\beta}\partial_\alpha x^\mu(u_0,u_1)\partial_\beta x_\mu(u_0,u_1), \qquad (u_0,u_1) = (\tau,\sigma), \tag{B.1}$$

where $x^\mu(\tau,\sigma)$ – string coordinates given in d-dimension space-time ($\mu = 0, 1, 2, \ldots, d-1$), $g_{\alpha\beta}(u_0,u_1)$ – is a second-rank metric tensor on the string surface (two dimensional Riemannian space u_0, u_1).

Now let us consider the Hamiltonian scheme which is based on the Arnowitt–Deser–Misner parametrization of metric tensor $g_{\alpha\beta}$ [46]

$$g_{\alpha\beta} = \Omega^2\begin{pmatrix} N_0^2 - N_1^2 & N_1 \\ N_1 & -1 \end{pmatrix}, \quad g^{\alpha\beta} = \frac{1}{\Omega^2 N_0^2}\begin{pmatrix} 1 & N_1 \\ N_1 & N_1^2 - N_0^2 \end{pmatrix}, \quad \sqrt{-g} = \Omega^2 N_0 \tag{B.2}$$

with the conformal invariant interval

$$ds^2 = g_{\alpha\beta}du^\alpha du^\beta = \Omega^2[N_0^2 d\tau^2 - (d\sigma + N_1 d\tau)^2], \tag{B.3}$$

where N_0 and $N_1(\tau,\sigma)$ are known in GR as the lapse function and "shift vector", respectively (compare formulas (5)).

The action (B.1) after the substitution (B.2) does not depend on the conformal factor Ω and takes the form

$$S = -\frac{\gamma}{2}\int_{\tau_1}^{\tau_2} d\tau\int_0^l d\sigma\left[\frac{(\dot{x}_\mu - N_1 x'_\mu)^2}{N_0} - N_0 x'^2\right], \tag{B.4}$$

where $\dot{x}_\mu = \partial_\tau x_\mu$, $x'_\mu = \partial_\sigma x_\mu$ and $\dot{x}_\mu - N_1 x'_\mu = Dx_\mu$ is the covariant derivative with respect to the metric (B.3). The action (B.4), the metric (B.3) and the covariant derivative Dx_μ are invariant under the "kinemetric" transformation $\tau \to \tilde{\tau} = f_1(\tau)$, $\sigma \to \tilde{\sigma} = f_2(\tau,\sigma)$ that are similar to group of transformation in GR (27), (28). The "kinemetric" transformations of the

differentials $d\tilde{\tau} = \dot{f}_1(\tau)d\tau,\ d\tilde{\sigma} = \dot{f}_2(\tau,\sigma)d\tau + f_2{}'(\tau,\sigma)d\sigma$ correspond to transformations of the string coordinates (compare with (28) in the text)

$$\begin{aligned} x_\mu{}'(\tau,\sigma) &= \tilde{x}_\mu{}'(\tilde{\tau},\tilde{\sigma}) f_2{}'(\tau,\sigma), \\ \dot{x}_\mu(\tau,\sigma) &= \dot{\tilde{x}}_\mu(\tilde{\tau},\tilde{\sigma})\dot{f}_1(\tau) + \tilde{x}_\mu{}'(\tilde{\tau},\tilde{\sigma})\dot{f}_2(\tau,\sigma), \\ N_0(\tau,\sigma) &= \tilde{N}_0(\tilde{\tau},\tilde{\sigma})\frac{\dot{f}_1(\tau)}{f_2{}'(\tau,\sigma)}, \quad N_1(\tau,\sigma) = \tilde{N}_1(\tilde{\tau},\tilde{\sigma})\frac{\dot{f}_1}{\tilde{f}_2{}'} + \frac{\dot{f}_2}{\tilde{f}_2{}'}, \end{aligned} \tag{B.5}$$

$$D_\tau x_\mu(\tau,\sigma) = \dot{f}_1(\tau) D_{\tilde{\tau}}\tilde{x}_\mu(\tilde{\tau},\tilde{\sigma}) \tag{B.6}$$

The variation S with respect to N_0 and N_1 leads to equation for determination N_0, N_1

$$\frac{\delta S}{\delta N_0} = \frac{(Dx_\mu)^2}{N_0^2} + x'^2 = 0 \ \Rightarrow N_0^2 = \frac{(\dot{x}x')^2 - \dot{x}^2 x'^2}{(x'^2)^2}, \tag{B.7}$$

$$\frac{\delta S}{\delta N_1} = \frac{(x'_\mu D x^\mu)}{N_0} \ \Rightarrow N_1 = \frac{(\dot{x}x')}{x'^2}, \tag{B.8}$$

The substitution of these equation in action (B.4) converts it into the standard Nambu–Goto action of the relativistic string [44]

$$S = -\gamma \int_{\tau_1}^{\tau_2} d\tau \int_0^l d\sigma \sqrt{(\dot{x}x')^2 - \dot{x}^2 x'^2}. \tag{B.9}$$

One can construct the Hamiltonian form of S. The conjugate momenta are determined by

$$p_\mu = \frac{\delta S}{\delta \dot{x}^\mu} = \gamma\frac{\dot{x}_\mu - N_1 x_\mu{}'}{N_0} \Rightarrow \dot{x}_\mu = N_0\frac{p_\mu}{\gamma} + N_1 x_\mu{}' \tag{B.10}$$

and density of Hamiltonian is obtained by the Legendre transformation

$$H = p_\mu \dot{x}^\mu - L = N_0\frac{P^2 + \gamma^2 x'^2}{2\gamma} + N_1(px'), \tag{B.11}$$

then

$$S = -\int_{\tau_1}^{\tau_2} d\tau \int_0^l d\sigma \left[p_\mu \dot{x}^\mu - H\right]. \tag{B.12}$$

The secondary class constraints arises by varying S with respect to N_0, N_1

$$\frac{\delta S}{\delta N_0} = \frac{p^2 + \gamma^2 x'^2}{2\gamma} = 0, \quad \frac{\delta S}{\delta N_1} = (px') = 0. \tag{B.13}$$

The equations of motion take the form

$$\begin{aligned} \frac{\delta S}{\delta x^\mu} &= \dot{p}_\mu - \frac{\partial}{\partial\sigma}(\gamma N_0 x'_\mu + N_1 p_\mu) = 0, \\ \frac{\delta S}{\delta p^\mu} &= \dot{x}_\mu - N_0\frac{p_\mu}{\gamma} + N_1 x'_\mu = 0. \end{aligned} \tag{B.14}$$

The standard gauge-fixing method is to fix the second class constraints (orthonormal gauge) $N_0 = 1, N_1 = 0$. In this case the equations of motion (B.14) reduce to d'Alambert ones for x_μ

$$\dot{p}_\mu = \gamma x_\mu{}'', \quad \dot{x}_\mu = \frac{p_\mu}{\gamma} \Rightarrow \ddot{x}_\mu - x_\mu{}'' = 0, \tag{B.15}$$

the conformal interval (B.3) takes the usual form

$$ds^2 = \Omega^2[d\tau^2 - d\sigma^2],$$

but the Hamiltonian (B.11) in view of the constraint (B.13) is equal to zero ($H = 0$).

There is another way to introduce evolution parameter as the object reparameterizations in the theory being adequate to the initial "kinemetric" invariant system and to construct the non-zero Hamiltonian. Accordingly to (39) we identify this evolution parameter with the time-like variable of the "center of mass" (CM) of a string defined as the total coordinate

$$X_\mu(\tau) = \frac{1}{l}\int_0^l x_\mu(\tau,\sigma)d\sigma. \tag{B.16}$$

Therefore, the reduction requires to separate the "center of mass" variables before variation of the action (B.4) which after substitution

$$x_\mu = X_\mu(\tau) + \xi_\mu(\tau,\sigma),\ x'_\mu(\tau,\sigma) = \xi'_\mu(\tau,\sigma), \tag{B.17}$$

$$\int_0^l \xi_\mu(\tau,\sigma) = 0 \tag{B.18}$$

takes the form

$$S = \frac{\gamma}{2}\int_{\tau_1}^{\tau_2} d\tau \left\{ \dot{X}^2(\tau)\int_0^l \frac{d\sigma}{N_0(\tau,\sigma)} + 2\dot{X}^\mu(\tau)\int_0^l d\sigma\left(\frac{\dot{\xi}_\mu - N_1\xi'_\mu}{N_0}\right) + \right.$$
$$\left. + \int_0^l d\sigma\left[\frac{(\dot{\xi}_\mu - N_1\xi'_\mu)^2}{N_0} - N_0\xi'^2(\tau,\sigma)\right]\right\}. \tag{B.19}$$

The usual determination of the conjugate momenta

$$P_\mu(\tau) = \frac{\delta S}{\delta \dot{X}^\mu(\tau)} = \gamma\dot{X}_\mu(\tau)\int_0^l \frac{d\sigma}{N_0(\tau,\sigma)} + \gamma\int_0^l d\sigma\left(\frac{\dot{\xi}_\mu(\tau,\sigma) - N_1\xi'_\mu(\tau,\sigma)}{N_0(\tau,\sigma)}\right), \tag{B.20}$$

$$\pi_\mu(\tau,\sigma) = \frac{\delta S}{\delta\dot{\xi}^\mu(\tau,\sigma)} = \gamma\dot{X}_\mu(\tau)\frac{1}{N_0(\tau,\sigma)} + \gamma\frac{\dot{\xi}_\mu(\tau,\sigma) - N_1\xi'_\mu(\tau,\sigma)}{N_0(\tau,\sigma)} \tag{B.21}$$

leads to the contradiction because $P_\mu(\tau)$ and $\pi_\mu(\tau,\sigma)$ are not independent , namely $\int_0^l \pi_\mu(\tau,\sigma)d\sigma = P_\mu(\tau)$ (compare (47), (48), (55)).

Thus for the consistent definition of the momentum of "center-mass" $P_\mu(\tau)$ and momentum of intrinsic variables $\pi_\mu(\tau,\sigma)$ we have to put strong constraint in action (B.19) (compare (53))

$$\int d\sigma\left(\frac{\dot{\xi}_\mu(\tau,\sigma) - N_1\xi'_\mu(\tau,\sigma)}{N_0}\right) = 0. \tag{B.22}$$

Then we obtain the following form of the reduced action:

$$S_{\rm red} = \frac{\gamma}{2}\int d\tau\left\{\dot{X}^2(\tau)\frac{l}{N(\tau)} + \int_0^l d\sigma\left[\frac{[\dot{\xi}_\mu(\tau,\sigma) - N_1\xi'_\mu(\tau,\sigma)]^2}{N_0(\tau,\sigma)} - N_0(\tau,\sigma){\xi'_\nu}^2(\tau,\sigma)\right]\right\}, \tag{B.23}$$

where $N(\tau)$ is the global lapse function

$$\frac{1}{N(\tau)} = \frac{1}{l}\int_0^l \frac{d\sigma}{N_0(\tau,\sigma)} = \langle N_0^{-1}\rangle. \tag{B.24}$$

For global momenta (compare with (47)) we get

$$P_\mu(\tau) = \frac{\partial S_{\rm red}}{\partial \dot X^\mu(\tau)} = \gamma \dot X_\mu(\tau) \frac{l}{N(\tau)} \tag{B.25}$$

and for the local (intrinsic) momenta (compare with (48))

$$\pi_\mu(\tau,\sigma) = \frac{\partial S_{\rm red}}{\partial \dot\xi^\mu(\tau,\sigma)} = \gamma \frac{\dot\xi_\mu(\tau,\sigma) - N_1(\tau,\sigma)\xi'_\mu(\tau,\sigma)}{N_0(\tau,\sigma)} \tag{B.26}$$

with two strong constraints (B.18) and (B.22)

$$\int_0^l \xi_\mu(\tau,\sigma) d\sigma = 0, \qquad \int_0^l \pi_\mu(\tau,\sigma) d\sigma = 0. \tag{B.27}$$

This separation conserves the group of the "kinemetric" transformation (B.5) and leads to Hamiltonian form of reduced action, in view of

$$\dot X_\mu(\tau) = \frac{N(\tau)}{l}\frac{P_\mu(\tau)}{\gamma}; \qquad \dot\xi_\mu(\tau,\sigma) = \frac{N_0(\tau,\sigma)}{\gamma}\pi_\mu(\tau,\sigma) + N_1(\tau,\sigma)\xi'_\mu(\tau,\sigma), \tag{B.28}$$

we get

$$H = P_\mu \dot X^\mu(\tau) + \pi_\mu \dot\xi^\mu - L = \frac{1}{2\gamma}\left\{\frac{N_0(\tau,\sigma)}{l}P^2(\tau) + N_0(\tau,\sigma)[\pi^2 + \gamma^2\xi'^2] + 2\gamma N_1(\pi\xi')\right\}, \tag{B.29}$$

$$S = \int_{\tau_1}^{\tau_2} d\tau \left\{P_\mu(\tau)\dot X^\mu(\tau) - N(\tau)\frac{P^2(\tau)}{2\gamma l} + \int_0^l d\sigma \left[(\pi\dot\xi) - N_0 \frac{\pi^2+\gamma^2\xi'^2}{2\gamma} - N_1(\pi\xi')\right]\right\}. \tag{B.30}$$

The equation of motion is split into global one

$$\frac{\delta S}{\delta \dot X^\mu(\tau)} = \dot P_\mu(\tau) = 0, \qquad \frac{\delta S}{\delta P^\mu(\tau)} = \dot X_\mu(\tau) - N(\tau)\frac{P_\mu}{\gamma l} = 0$$

and local one

$$\frac{\delta S}{\delta \pi^\mu(\tau,\sigma)} = \dot\xi_\mu(\tau,\sigma) - \frac{N_0}{\gamma}\pi_\mu - N_1\xi'_\mu = 0, \quad -\frac{\delta S}{\delta\xi^\mu(\tau,\sigma)} = \dot\pi_\mu - \frac{\partial}{\partial\sigma}(N_1\pi_\mu + \gamma N_0 \xi'_\mu) = 0. \tag{B.31}$$

The variation of the action (B.30) with respect to $N_0(\tau)$ results in the constraint

$$\frac{\delta S}{\delta N_0(\tau,\sigma)} = \frac{N^2(\tau)}{N_0^2(\tau,\sigma)}\frac{P^2(\tau)}{2\gamma l^2} + \frac{\pi^2(\tau,\sigma) + \gamma^2\xi'^2(\tau,\sigma)}{2\gamma} = 0, \tag{B.32}$$

here it is necessary to take into account that the variation over the global lapse function (B.24) leads to

$$\frac{\delta N(\tau)}{\delta N_0(\tau,\sigma)} = \frac{N^2(\tau)}{N_0^2(\tau,\sigma)}. \tag{B.33}$$

The variation of the (B.30) with respect to $N_1(\tau,\sigma)$ results in the constraint for local variables

$$\frac{\delta S}{\delta N_1(\tau,\sigma)} = (\pi\xi'(\tau,\sigma)) = 0. \tag{B.34}$$

Now we introduce the Hamiltonian density for local excitations

$$\mathcal{H}(\tau,\sigma) = -\frac{\pi^2(\tau,\sigma)+\gamma^2\xi'^2(\tau,\sigma)}{2\gamma} \tag{B.35}$$

and rewrite (B.32) in the form

$$\frac{N(\tau)}{N_0(\tau,\sigma)l}\sqrt{P_\mu^2(\tau)} = \sqrt{2\gamma\mathcal{H}(\tau,\sigma)}. \tag{B.36}$$

One integrates (B.36) over σ and taking into account the normalization equality (B.24)

$$\frac{1}{l}\int_0^l \frac{N(\tau)}{N(\tau,\sigma)}d\sigma = 1, \tag{B.37}$$

it leads to global constraint

$$M_{\rm st} = \sqrt{P_\mu^2(\tau)} = \int_0^l \sqrt{2\gamma\mathcal{H}(\tau,\sigma)}d\sigma = l\langle\sqrt{2\gamma\mathcal{H}}\rangle, \tag{B.38}$$

where $M_{\rm st}$ is mass of the string. The local part of the constraint (B.36) can be obtained by substitution (B.38) into (B.36)

$$\frac{N(\tau)}{N_0(\tau,\sigma)} = \frac{\sqrt{\mathcal{H}(\tau,\sigma)}}{\langle\sqrt{\mathcal{H}}\rangle}, \tag{B.39}$$

which coincide with the equation (37) in the text. Finally, after substitution (B.38), (B.39) into action (B.30) we can derive the constraint-shell action, whereas the equation $N(\tau)P^2(\tau)/2\gamma l = \int N_0(\tau,\sigma)\mathcal{H}(\tau,\sigma)d\sigma$

$$S_{\rm const-shell} = \int_{\tau_1}^{\tau_2} d\tau\{P_\mu\dot{X}^\mu(\tau) + \int_0^l d\sigma[\pi_\mu(\tau,\sigma)\dot{\xi}^\mu(\tau,\sigma) - N_1(\tau,\sigma)\pi_\mu(\tau,\sigma)\xi'_\mu(\tau,\sigma)]\}. \tag{B.40}$$

Again the variation with respect to $N_1(\tau,\sigma)$ results in subsidiary condition $(\pi_\mu(\tau,\sigma)\xi'^\mu(\tau,\sigma)) = 0$. Now in the "center-mass" coordinate system $P_i(\tau) = 0$, $\;P_0(\tau) = \int_0^l d\sigma\sqrt{2\gamma\mathcal{H}(\tau,\sigma)}$ the action (B.40) takes the form

$$S_{\rm const-shell} = \int_{\tau_1}^{\tau_2} d\tau \int_0^l d\sigma\{\pi_\mu(\tau,\sigma)\dot{\xi}^\mu(\tau,\sigma) + \sqrt{2\gamma\mathcal{H}(\tau,\sigma)}\dot{X}_0(\tau)\}, \tag{B.41}$$

which describes the dynamics of a local (intrinsic) canonical variables of a string with non-zero Hamiltonian because (B.41) can be rewritten in the form

$$S_{\rm const-shell} = \int_{x_1}^{x_2} dX_0 \int_0^l d\sigma\left\{\pi_\mu(X_0,\sigma)\frac{\partial\xi^\mu(X_0,\sigma)}{\partial X_0} + \sqrt{2\gamma\mathcal{H}(X_0,\sigma)}\right\}, \tag{B.42}$$

where

$$2\gamma\mathcal{H}(X_0,\sigma) = -[\pi^2(X_0,\sigma)+\gamma^2\xi'^2(X_0,\sigma)]$$

and time $dX_0 = \dot{X}_0(\tau)d\tau$ is invariant with respect to $d\tilde{\tau} = f_1 d\tau$.

In the gauge-fixing method, by using the kinemetric transformation, we have to put $N_0(\tau,\sigma) = 1$, $N_1(\tau,\sigma) = 0$ (this requirement does not contradict to Eq. (B.37) in view of Eqs. (B.38), (B.39)). Then according to [48]

$$\sqrt{\mathcal{H}(\tau,\sigma)} = \frac{1}{l}\int_0^l d\sigma\sqrt{\mathcal{H}(\tau,\sigma)} = \frac{M_{\rm st}}{\sqrt{2\gamma}\,l} \tag{B.43}$$

means that the Hamiltonian $\sqrt{2\gamma\mathcal{H}(\tau,\sigma)}$ is constant. In this case the equations for the local variables obtained by varying the action (B.42) take the form

$$\frac{\delta S}{\delta\pi^{\mu}(X_0,\sigma)} = \frac{\partial\xi_{\mu}(X_0,\sigma)}{\partial X_0} - \frac{\pi_{\mu}(X_0,\sigma)}{\sqrt{-(\pi^2+\gamma^2\xi'^2)}} = 0, \tag{B.44}$$

$$\frac{\delta S}{\delta\xi^{\mu}(X_0,\sigma)} = -\frac{\partial\pi_{\mu}(X_0,\sigma)}{\partial X_0} + \gamma^2\frac{\partial/\partial\sigma\xi'_{\mu}(X_0,\sigma)}{\sqrt{-(\pi^2+\gamma^2\xi'^2)}} = 0. \tag{B.45}$$

If we put $\sqrt{-(\pi^2+\gamma^2\xi'^2)} = \gamma$, $(M_{\mathrm{st}} = \gamma l)$, then it leads again to d'Alambert equation for $\xi_{\mu}(X_0,\sigma)$

$$\frac{\partial^2\xi_{\mu}(X_0,\sigma)}{\partial X_0^2} = \frac{\partial^2\xi_{\mu}(X_0,\sigma)}{\partial\sigma^2}. \tag{B.46}$$

The general solution of these equations in class of functions (B.27) with boundary conditions for the open string ${\xi'}_{\mu}(X_0,0) = {\xi'}_{\mu}(X_0,l) = 0$ is given as usually by the Fourier series

$$\begin{aligned}
\xi_{\mu}(X_0,\sigma) =& \frac{1}{2\sqrt{\gamma}}[\Psi_{\mu}(X_0+\sigma) + \Psi_{\mu}(X_0-\sigma)],\\
{\xi'}_{\mu}(X_0,\sigma) =& \frac{1}{2\sqrt{\gamma}}[{\Psi'}_{\mu}(X_0+\sigma) - {\Psi'}_{\mu}(X_0-\sigma)],\\
\pi_{\mu}(X_0,\sigma) =& \gamma\frac{\partial\xi_{\mu}(X_0,\sigma)}{\partial X_0} = \frac{\sqrt{\gamma}}{2}[{\Psi'}_{\mu}(X_0+\sigma) + {\Psi'}_{\mu}(X_0-\sigma)],
\end{aligned} \tag{B.47}$$

where

$$\Psi_{\mu}(z) = i\sum_{n\neq 0}\frac{\alpha_{n\mu}}{n}e^{-\frac{i\pi n}{l}z}, \quad {\Psi'}_{\mu}(z) = \frac{\pi}{l}\sum_{n\neq 0}\alpha_{n\mu}e^{-\frac{i\pi n}{l}z} \tag{B.48}$$

(it does not contain zero harmonics $(n\neq 0)$). The substitution of ξ_{μ} and π_{μ} in this form into (B.35) and taken into consideration (B.43) leads to density of Hamiltonian

$$\mathcal{H} = -\frac{1}{4}[\Psi'^2_{\mu}(X_0+\sigma) + \Psi'^2_{\mu}(X_0-\sigma)] = \frac{M_{\mathrm{st}}^2}{2\gamma l^2}. \tag{B.49}$$

For the constraint $(\pi\xi') = 0$ in terms of the vector ${\Psi'}_{\mu}$ (B.47) we obtain

$$\pi_{\mu}(X_0,\sigma)\xi'^{\mu}(X_0,\sigma) = \frac{1}{4}[\Psi'^2_{\mu}(X_0+\sigma) - \Psi'^2_{\mu}(X_0-\sigma)] = 0, \tag{B.50}$$

then from (B.49) and (B.50) finally we get

$$\Psi'^{\,2}_{\mu}(X_0+\sigma) = \Psi'^{\,2}_{\mu}(X_0-\sigma) = -\frac{M_{\mathrm{st}}^2}{l^2\gamma}. \tag{B.51}$$

It means that $\Psi'_{\mu}(z)$ is the modulo-constraint space-like vector and in terms of its representation (B.48) the equalities (B.51) can be rewritten

$$-\Psi'^{\,2}_{\mu}(z) = \frac{\pi^2}{l^2}\sum_{k=-\infty}^{\infty}L_k e^{-i\frac{\pi k}{l}z} = \frac{M_{\mathrm{st}}^2}{l^2\gamma}, \tag{B.52}$$

where

$$L_k = -\sum_{n\neq k,0}\alpha_{n\mu}\alpha^{\mu}_{k-n}, \qquad L_k^* = L_{-k}\ .$$

Now we can see that the zero harmonic of this constraint determines the mass of a string

$$M_{\rm st}^2 = \pi^2 \gamma L_0 = -\pi\gamma \sum_{n\neq 0} \alpha_{n\mu}\alpha^{\mu}_{-k}, \qquad \alpha^{\mu}_{-k} = \alpha^{*\mu}_{k} \tag{B.53}$$

and coincides with standard definition in string theory [44], however the non-zero harmonics of constraint (B.52)

$$L_{k\neq 0} = -\sum_{n\neq 0,k} \alpha_{nk}\alpha^{\mu}_{k-n} = 0 \tag{B.54}$$

strongly differ from the standard theory because they do not depend on the global motion(do not depend on P_μ) and do not contain the interference term $P_\mu \Psi^{\mu\prime}$, because of our constraint (B.51) we can rewrite

$$l^2\gamma {\Psi'_\mu}^2(z) + P_\mu^2 = 0 \tag{B.55}$$

instead standard one [44]

$$(P_\mu + l\sqrt{\gamma}\Psi'_\mu(z))^2 = 0. \tag{B.56}$$

Therefore our approach coincides with the Röhrlich one [47], which is based on the gauge condition $P_\mu \xi^\mu = 0, \quad P_\mu \pi^\mu = 0 \Rightarrow P_\mu \alpha^\mu_n = 0, \quad n \neq 0$. One use of that condition for eliminating the time components ξ_0, π_0 being constructed in the frame of reference ($P_i = 0$) leads to formula (B.47)–(B.54), where Ψ'_0 and α_{n0} are equal to zero.

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